A NEW LOOK AT THE FORWARD PREMIUM PUZZLE

Jacob Boudoukh,[a] Matthew Richardson,[b] and Robert F. Whitelaw[b*]

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Abstract

The forward premium anomaly, i.e., the empirical evidence that exchange rate changes are negatively related to interest rate differentials, is one of the most robust puzzles in financial economics. We add to this literature by recasting the underlying parity relation in terms of cross-country differences between forward interest rates rather than spot interest rates. The differences using spot and maturity-matched forward rates are dramatic. As the maturity of the forward interest rate differential increases, the anomalous sign on the coefficient in the traditional specification is reversed, and the explanatory power increases. We present a simple model of interest rates, inflation, and exchange rates that explains this novel empirical evidence. The model is based on interest rate distortions due to Taylor rules and exchange rate determination involving not just purchasing power parity, but also effects due to real rate differentials and subsequent reversion of the exchange rate to fundamentals. We develop and test additional implications of this model. A key finding is that the effect of current interest rate differentials on exchange rates can be decomposed into two offsetting components, which, if used separately, greatly increase the explanatory power of regression models for exchange rates.

[*a]Arison School of Business, IDC; [b]Stern School of Business, NYU and NBER. We thank Philippe Jorion for providing us with term structure data. We are grateful to Thomas Philippon, Dave Backus and seminar participants at NYU, NYU Salomon Center, Research Conference for Corporate Associates, Tel Aviv University, IDC, the University of Virginia, Rutgers University, Baruch College, the Swiss Society for Financial Market Research, the Western Finance Association Meetings, particularly the discussant Greg Duffee, and the NHH Conference in Memoriam of Jan Mossin on Asset Allocation, Investments, and Asset Pricing, particularly the discussant Magnus Dahlquist, for many helpful comments. Contact: Prof. R. Whitelaw, NYU, Stern School of Business, 44 W. 4th St., Suite 9-190, New York, NY 10012; *rwhitela@stern.nyu.edu*.

I. Introduction

Well over one hundred papers document, in some form or another, the forward premium anomaly—namely, that future exchange rate changes do not move one-for-one with interest rate differentials across countries. In fact, they tend to move in the opposite direction (e.g., see Hodrick (1987) and Engel (1996) for survey evidence). This anomaly has led to a plethora of papers over the last two decades that develop possible explanations with only limited success. It is reasonable to conclude that the forward premium anomaly is one of the more robust puzzles in financial economics. Parallel to work on the forward premium puzzle, another literature has developed, starting with Meese and Rogoff (1983), documenting an equally startling puzzle: exchange rates do not seem to be related to fundamentals.[1] The random walk model has proven almost unbeatable, even against models with a variety of finance and macro variables.

This paper looks at the forward premium anomaly, and the fundamental determinants of exchange rates, in a novel way by recasting the uncovered interest rate parity (UIP) relation in terms of future exchange rate movements against forward interest rate differentials across countries. We study three currencies, the U.S. dollar, the British pound, and the German mark/Euro, over the 1976-2010 sample period. In stark contrast to current research on uncovered interest rate parity, past forward interest rate differentials have strong forecasting power for exchange rates. R^2s at some horizons exceed 10% for annual exchange rate changes relative to about 2% for the traditional specification. Moreover, the direction of these forecasts coincides with the theoretical implications of UIP.

We present a simple reduced-form model of interest rates, inflation, and exchange rates that fits the contrasting empirical evidence on UIP when using forward, rather than spot, interest rate differentials. Though the model is reduced-form in nature, it is developed to capture existing stylized facts. Exchange rates are determined by three components: (i) purchasing power parity (PPP), (ii) real rate differentials arising from interest rate distortions due to the application of Taylor rules, and (iii) a positive probability that the currency will revert to PPP. The model can jointly explain why uncovered interest rate parity fails, why it appears to work better using lagged forward interest rate differentials, and why the explanatory power for exchange rates increases with the horizon, i.e., more lagged and stale information. The key insight is that, while real rate differentials do lead to PPP violations and the rejection of UIP, the build-up of these violations generally gets reversed, which we model as a "crash" back to PPP.

Two additional implications of this model are developed and tested. First, a key theoretical finding of our model is that current interest rate differentials can be decomposed into two main offsetting components, which, if used separately, increase the explanatory power for exchange rates. This

[1] Meese and Rogoff (1983) find that the literature's typical structural models of exchange rates cannot outperform a naïve random walk model even when one uses ex-post values of the variables of interest such as money supply, real income, inflation and interest rates. These findings are revisited and confirmed by Cheung, Chinn, and Garcia Pascual (2003) using updated data. For a theoretical analysis of this issue, see Engel and West (2005).

implication is tested and confirmed in the data. For example, by decomposing current interest rate differentials into forecasts, under the expectations hypothesis, based on prior forward interest rate differentials and corresponding forecast errors, the R^2s jump from 2% for the UIP regressions to between 14% and 16% for the USD/GBP exchange rate and between 6% and 13% for the US/DEM exchange rate. Most important, the coefficients on the two components of the interest rate differential are of opposite signs. The intuition is that the forward interest rate differential has better information about the magnitude of the deviation from PPP, and thus the impact of a currency crash, than does the current interest rate differential. In contrast, the forecast error better captures the current real rate differential and the associated deviations from PPP in exchange rates.

A second implication of the model is that the change in exchange rates is a function of two key state variables—the interest rate differential and the magnitude of the deviation of the current exchange rate from that implied by PPP. These two variables identify the same decomposition discussed above, with the interest rate differential capturing violations of UIP associated with real rate distortions and the deviation from PPP capturing the reversal of this effect in the longer term as exchange rates revert to fundamentals. The deviation of the exchange rate from PPP is not directly observable, but we can calculate the real exchange rate, which, up to a constant, captures the same information in the context of our model. Thus, we regress annual exchange rate changes on the interest rate differential and the real exchange rate. The results are striking and consistent with the theory. Controlling for the real exchange rate, the coefficient on the interest rate differential becomes more negative and is identified more precisely. Moreover, together the variables generate R^2s of approximately 25% for both currencies.

This paper is organized as follows. Section II introduces the data and presents new empirical evidence on the exchange rate parity relation in terms of forward interest rate differentials. In Section III, we present our reduced form model of exchange rates, which can explain this new evidence. Of particular importance, we derive two new testable implications of the model: (1) a decomposition of predictable exchange rate changes generated by factoring interest rate differentials into their expected component, based on lagged forward interest rate differentials, and the associated innovation, and (2) a related decomposition that uses interest rate differentials and deviations of exchange rates from PPP. In Section IV, we provide additional empirical evidence in support of the model in the context of these two new results. Section V concludes.

II. Uncovered Interest Rate Parity: Evidence

A. Data

We use monthly data from Datastream on exchange rates, inflation rates, and interest rates for the United States, the United Kingdom, and Germany from January 1976 to August 2010**,** a total of 416 months. Our sample is limited to these three countries due to the necessity of having term structure data at annual maturities out to five years. Data for exchange rates (the Euro was substituted for the Deutschmark in the latter part of the sample) and CPI levels are available for the full period. Data for the term structure of zero-coupon interest rates are derived from LIBOR data (with maturities of six and twelve months) and swap rates (two-, three-, four-, and five-year semi-annual swap rates).[2] Since swap data only become available in the late 1980s, we augment our zero curve data with data from Philippe Jorion. Jorion and Mishkin (1991) collect and derive data for zero coupon bonds from one month to five years for the United States, the United Kingdom, and Germany from August 1973 to December 1988.[3] Swap and LIBOR data is preferred to typical government bond data because the quotes are more liquid and less prone to missing data, supply and demand effects, and tax-related biases. To the extent that there is a swap spread (i.e., the difference between the swap and government bond rates) embedded in the data, its effect is diminished in our analysis by our use of interest rate differentials across countries.

Using the exchange rate and CPI data described above, we compute annual changes in log exchanges rates with the U.S. dollar as the base currency, i.e., we examine changes in the USD/GBP and USD/DEM rates, and annual log changes in the CPI indexes, i.e., the inflation rates for the three countries. We also combine these series to construct real exchange rates for both country pairs. Given the monthly frequency of the underlying data, adjacent annual changes have an 11-month overlap. Using the zero curve data, we compute continuously compounded one-year spot interest rates and one-year forward interest rates for each country from years 1 to 2, 2 to 3, 3 to 4, and 4 to 5. Our analysis matches the j to $j+1$ year forward interest rate at time $t-j$ with the subsequent exchange rate change from time t to time $t+1$.[4] To ensure that we use exactly the same exchange rate series for all regressions, we eliminate the first four years of the exchange rate data, truncating the interest rate series accordingly. The final dataset consists of annual exchange rate changes, with the first observation corresponding to calendar year 1980 and the last to the period September 2009 to August 2010 (357 observations sampled monthly), spot interest rates from 1/1980-9/2009, and forward interest rates over the periods 1/1979–9/2008, 1/1978–9/2007, 1/1977–9/2006, and 1/1976–9/2005 for horizons $j = 1,\ldots,4$, respectively (all with 357 observations). Table 1, Panels A and B contain descriptive statistics for these variables.

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[2] Cubic spline functions are fitted each month for each country to create a zero curve for maturities of 6, 12, 18, …, 60 months. Our spline function fits the available data exactly, namely LIBOR rates for the 6-month and 12-month maturities, and semi-annual swap rates for maturities of 24 months, 36 months, 48 months, and 60 months. Therefore, the only maturities we need to spline are 18 months, 30 months, 42 months and 54 months. We maximize the smoothness of the spline function over these unknowns by minimizing the sum of squared deviations.

[3] We thank Philippe Jorion for graciously providing us with the data.

[4] Throughout the paper we use annual exchange rate changes and annual interest rates and forward rates; thus, for ease of exposition, all periods are denoted in years. However, as noted above, these annual quantities are calculated on a monthly overlapping basis to maximize the information content of the empirical analysis.

B. Existing Evidence

The expectations hypothesis for exchange rates (forward parity) is commonly written as

$$E_t s_{t+j} = f_t^{\,j}, \tag{1}$$

where s_{t+j} is the log of the spot price of foreign currency at time *t+j,* and $f_t^{\,j}$ is the log of the *j*-year forward exchange rate at time *t*. Assuming no arbitrage and covered interest rate parity (i.e., $f_t^{\,j} - s_t = j(i_{t,j} - i_{t,j}^*)$, where $i_{t,j}$ is the domestic, *j*-year, continuously compounded (log), annualized interest rate at time *t* and the superscript * denotes the corresponding foreign interest rate), the expected change in the exchange rate equals the interest rate differential. Thus, one standard way of testing equation (1) for annual changes in exchange rates is to estimate the regression

$$\Delta s_{t,t+1} = \alpha + \beta(i_{t,1} - i_{t,1}^*) + \varepsilon_{t,1}, \tag{2}$$

where $\Delta s_{t,t+1} \equiv s_{t+1} - s_t$. Under uncovered interest rate parity (UIP), α and β should be 0 and 1, respectively. That is, high interest rate currencies should depreciate and low interest rate currencies should appreciate in proportion to the interest rate differential across the countries. Intuitively, expected (real) returns on bonds in the two countries should be equal. This hypothesis has been resoundingly rejected, and, most alarming, β tends to be negative, i.e., exchange rates move in the opposite direction to that implied by the theory. In the context of equation (1), the forward premium, $s_{t+j} - f_t^{\,j}$, has a systematic bias and is predictable.[5]

One possible explanation for these findings is the existence of a risk premium in exchange rates. However, in order for this omitted variable in the regression in equation (2) to cause the coefficient β to change signs, this risk premium must exhibit significant time-variation and be negatively correlated with the interest rate differential, as noted in Fama (1984). While such a risk premium could explain the results from a statistical perspective, from an economic standpoint the key challenge is to identify for what risk this premium is providing compensation. So far, attempts to match the implied risk premium to economic risks have proven unsuccessful.[6]

As a first look at equation (2), Table 1, Panel C reports estimates from regressions of annual exchange rate changes on interest rate differentials on a monthly overlapping basis. The β coefficients (t-statistics for β = 1) are –0.82 (2.07) and –0.72 (2.39) for the USD/GBP and USD/DEM, respectively, confirming the well-known negative relation between exchange rates and interest rate differentials. While

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[5] See, e.g., Engel (1996) and Lewis (1995) for surveys of this literature. Interestingly, some evidence suggests that the forward premium anomaly may be confined to developed economies and may be asymmetric or state dependent even in those economies (Bansal and Dahlquist (2000), Wu and Zhang (1996)).

[6] See, e.g., Bekaert (1996), Mark and Wu (1998) and Graveline (2006).

the estimates are fairly noisy, tests of the null hypothesis that the coefficients equal 1 can be resoundingly rejected.

The low R^2s in the regressions are also notable, and this feature is both disappointing and puzzling. The key fundamentals underlying expected exchange rate movements are interest rate differentials between countries. These interest rate differentials, in theory, represent expected inflation rate differentials. Since inflation is fairly predictable (see, e.g., Fama and Gibbons (1984)), and inflation differentials are a fundamental driver of exchange rates via purchasing power parity, one would have expected the model to explain a much larger degree of the variation.

C. Information about Exchange Rate Changes in Long-Maturity Forward Rates

Equation (1), UIP, is almost always cast in terms of interest rate differentials and then tested using equation (2). In this subsection, we present a novel way to analyze UIP by recasting the parity relation in terms of future exchange rate movements against forward interest rate differentials across countries.

Specifically, we can also use equation (1) to define expected changes in future exchange rates as the difference between two forward exchange rates. That is,

$$E_t[\Delta s_{t+j,t+k}] = f_t^k - f_t^j, \tag{3}$$

where $k > j$. Under the expectations hypothesis of exchange rates, the period t expected depreciation from $t+j$ to $t+k$ equals the difference in the corresponding forward exchange rates at time t. Under covered interest rate parity, we can replace the forward exchange rates in equation (3) with the interest rate differentials between the two countries, i.e.,

$$E_t[\Delta s_{t+j,t+k}] = k\,(i_{t,k} - i^*_{t,k}) - j\,(i_{t,j} - i^*_{t,j}). \tag{4}$$

Rearranging the interest rate differential terms in equation (4), and using the definition of forward interest rates, we get

$$\begin{aligned} E_t[\Delta s_{t+j,t+k}] &= (k\,i_{t,k} - j\,i_{t,j}) - (k\,i^*_{t,k} - j\,i^*_{t,j}) \\ &= (k-j)(if_t^{\,j,k} - if_t^{\,j,k^*}) \end{aligned}, \tag{5}$$

where $if_t^{\,j,k}$ and $if_t^{\,j,k^*}$ are the continuously compounded, annualized, forward interest rates at time t from $t+j$ to $t+k$ for domestic and foreign currencies, respectively. Equation (5) is the basis for the empirical analysis to follow. It says that the expected depreciation in future exchange rates is equal what we call *the forward interest rate differential*.

Equation (5) extends the classical approach to characterizing and testing the expectations hypothesis presented in equations (1) and (2). It implies a more general specification of the expectation hypothesis,

$$\Delta s_{t,t+1} = \alpha + \beta(if_{t-j}^{j,j+1} - if_{t-j}^{j,j+1^*}) + \varepsilon_{t-j,j+1}. \quad (6)$$

Under the expectations hypothesis of exchange rates, the annual exchange rate change from t to t+1 should move one-for-one with the forward interest rate differential from j to j+1 that was set at time t–j. That is, α and β should equal 0 and 1 respectively. Equation (2) is a special case of equation (6) for j = 0.

Using regression equation (6), Table 2 provides estimates over different horizons and across different currencies for tests of the expectations hypothesis of exchange rates. In contrast to Table 1, Panel C and the conclusions in much of the literature, Table 2 shows that forward interest rate differentials can predict changes in future exchange rates. At least as important is that their predictive power has the right sign. The U.S./Germany forward interest rate differentials at horizons of one to four years yield coefficients of 0.65, 0.68, 2.01, and 3.17 for the USD/DEM exchange rate. The results for the USD/GBP exhibit a similar pattern, with coefficients of 0.96, 3.37, 1.95, and 2.56 looking forward one to four years, respectively. These results are quite different from the significant negative coefficients that plague Table 1, Panel C.

The coefficient estimates exhibit two apparent features in addition to the fact that they are positive. First, they tend to increase in the horizon. Second, for longer horizons they seem to exceed the theoretical value of 1. However, these coefficient estimates are noisy, especially at longer horizons, so more formal tests are warranted. As Table 2, Panel B reports, joint tests of the hypothesis that β = 1 at each horizon across the currencies yields only one rejection (j=2) and three non-rejections (j=1, 3, and 4) at the 5% level using the Wald test. None of the four Lagrange multiplier (LM) tests generate rejections at the 5% level, with p-values ranging from 20% to 95%.[7] Table 2, Panel B also documents tests across horizons for a given currency and across currencies. Testing the null hypothesis of β =1 across horizons j=1, 2, 3, and 4 (but not j=0), the LM test does not reject the null for either currency, and the Wald test rejects it for the USD/GBP, but with a p-value of 0.08, hence only at the 10% level. Across currencies, the Wald test again rejects, but the LM test does not. These results contrast starkly with those for the one-year horizon (j=0), which generate rejections of uncovered interest rate parity for both tests and a restricted coefficient estimate of –0.75.

Note that equation (6) exploits the information in the entire forward curve. However, the error term is now a j-step ahead forecast, and is serially correlated up to j observations. Therefore, one of the difficulties in studying multi-step ahead forecast regressions like those specified in equation (6) is the availability of data. While sophisticated econometrics have somewhat alleviated the problem (Hansen and

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[7] Throughout this study, we employ both the Lagrange multiplier and Wald statistics for testing the joint hypotheses. As shown by Berndt and Savin (1977), there is a numerical ordering between these statistics, which may lead to different inferences being drawn. For an especially relevant discussion, see Bekaert and Hodrick (2001) in the context of testing the expectations hypothesis of the term structure. In their context, the Wald test over-rejects while the Lagrange multiplier test under-rejects, results that are consistent with our simulation evidence discussed later.

Hodrick (1980) and Hansen (1982)), the benefits are still constrained by the number of independent observations. There are two sources for the serial correlation of the error term. The first arises from sampling annual exchange rate changes on a monthly basis, leading to a moving average structure out to 11 months. Sampling at the monthly frequency improves the efficiency of the estimators but only to a degree (Boudoukh and Richardson (1994) and Richardson and Smith (1989)). The second potential source arises directly from the *j*-step ahead forecast. For the regression in equation (6), however, the degree of serial correlation in the errors depends upon the relative variance of exchange rates versus interest rate differentials, and the correlation of unexpected shocks to these variables. There are strong reasons to suspect that these factors mitigate the serial correlation problem. Table 1, Panel A shows that exchange rates are much more variable than interest rate differentials and are relatively unpredictable. Therefore, because the forecast update component of the residual in equation (6) is likely to be small relative to the unpredictable component as we move forward in time, the induced serial correlation in the errors will be correspondingly small, and the overlap will not substantially reduce the effective number of independent observations. This intuition is confirmed through a Monte Carlo simulation described in Appendix A.

In Table 2, Panel A we also report statistics from the simulation model of Appendix A. We report the cross-sectional standard deviation (across replications) of the relevant parameter estimate (in the column "SD"), and the two-sided simulated P-value for the test that $\beta = 1$ (in the column "P-value"), i.e., the percentage of the replications in which the absolute magnitude of deviation of the estimated coefficient from one equals or exceeds the deviation for the estimated coefficient from the actual data. For these calculations, we simulate under the null hypothesis of $\beta = 1$ and use the resampled exchange rate changes for the relevant exchange rate, but simulating under normality produces similar results. The cross-sectional standard deviations tend to exceed the reported standard errors, especially at longer horizons, suggesting that these standard errors may be somewhat understated. However, the inferences drawn from the P-values are consistent with those from standard hypothesis test of the individual coefficients. Specifically, the short-horizon ($j=0$) coefficients are statistically significantly different from one, as is the coefficient for $j=2$ for the USD/GBP.

As a final comment on the evidence, note that in Table 2 the regression R^2s have a tendency to increase with the horizon. While the dependent variable, i.e., annual exchange rate changes, is the same, the forecasting variable differs. For the USD/GBP [USD/DEM], the R^2s are higher for three [two] out of four of the forward interest rate differential regressions (equation (6)) than for the interest rate differential regression (equation (2)). What is remarkable about this result is that the information in the former regressions is (i) old relative to current interest rates, and (ii) more subject to measurement error due to the calculation of forward rates. We argue below that this finding is an important clue to understanding the fundamental relation between exchange rates, inflation, and interest rates, and, more importantly, the forward premium anomaly.

III. A Simple Model of Exchange Rates and the Forward Premium Anomaly

The results provided in Section II are important stylized facts that need to be explained in the context of recent attempts at solving the forward premium puzzle of exchange rates. First, there is the need to reconcile the forward premium anomaly (i.e., a negative β in equation (2)) with the forward interest rate differential results (i.e., positive βs in equation (6)). Second, the coefficients in the forward interest rate differential regressions appear to increase in maturity and exceed the theoretical value of 1 for longer horizons. Third, the explanatory power of the forward interest rate differential also increases in the horizon over which the regressions are estimated, i.e., with information about countries' future interest rates that becomes increasingly stale.

In this section, we present a simple, reduced-form model of exchange rates, interest rates and inflation rates across countries. Though simple in structure, the model is built around assumptions consistent with the existing literature, and it can provide one potential explanation for the observed behavior of uncovered interest rate parity using spot and forward rate differentials.

A. A Simple, Reduced-Form Model of Exchange Rates

Our reduced-form model has four components, dealing with interest rates, inflation rates, forward rates of interest, and exchange rates. For simplicity, we focus on fundamentals related only to inflation rates, assuming that real growth across countries is constant. Also, for ease of exposition and without loss of generality, all variables are mean-adjusted, that is, we suppress all constants in the equations. We also assume symmetry between countries and focus on just two horizons, which we denote periods 1 and 2.

The first key feature of our model is how interest rates are formed in each country, and, in particular, the source of their "distortion" from fundamentals, in this case, from expectations about future inflation rates. While previous research has motivated such distortions in terms of risk or biased expectations, we choose to model it in terms of a Taylor (1993) rule in which the monetary authority of each country sets the short-term interest rate to temper inflation:

$$E_t[r_{t,t+1}] = i_{t,1} - E_t[\pi_{t,t+1}] = \gamma \pi_{t-1,t} \qquad \gamma \geq 0 , \tag{7}$$

where $E_t[r_{t,t+1}]$ is the expected 1-period real rate, $i_{t,1}$ is the 1-period nominal rate, and $\pi_{t-1,t}$ is the inflation rate (all in log form). In other words, when inflation is above its mean, the central bank increases interest rates, which leads, in expectation, to time-varying real rates that are proportional to the level of inflation. While based on a simple Taylor rule, this model is broadly consistent with recent more elaborate empirical specifications and tests of Taylor rules in the context of the exchange rate literature, including Engel and West (2006), Engel, Nelson, and West (2007), Clarida and Waldman (2007), and Mark (2009), among others.

The second feature of our model is an autoregressive process for inflation in each country. We propose a simple AR(1) model though more elaborate specifications could be modeled. The idea is that the application of the Taylor rule, combined with the underlying fundamentals of the economy, leads to:

$$\pi_{t,t+1} = \theta \pi_{t-1,t} + \varepsilon_{t,t+1} \quad . \tag{8}$$

The third feature of the model is the determination of long-term interest rates or, equivalently, forward rates. While there is empirical evidence of violations of the expectations hypothesis of interest rates (EHIR),[8] for simplicity we impose the expectations hypothesis in the model.[9] Specifically, we set the forward interest rate equal to the expectation of the future spot rate:

$$if_t^{1,2} = E_t[i_{t+1,1}], \tag{9}$$

where $if_t^{1,2}$ is the forward interest rate between *t+1* and *t+2* set at time *t*. Thus, forward rates anticipate any future distortions in spot rates associated with the Taylor rule specified in equation (7).

The final, and most important, feature of the model describes the evolution of exchange rates. Motivated by the existing literature, exchange rate changes are broken down into three pieces as follows:

$$\begin{aligned} \Delta s_{t,t+1} &= (\pi_{t,t+1} - \pi^*_{t,t+1}) \\ &\quad + \delta\left(E_{t+1}[r_{t+1,t+2}] - E_{t+1}[r^*_{t+1,t+2}]\right) \qquad . \\ &\quad + D_{t+1}\sum_{v=1}^{\tilde{W}}\left[-\delta\left(E_{t+2-v}[r_{t+2-v,t+3-v}] - E_{t+2-v}[r^*_{t+2-v,t+3-v2}]\right)\right] \qquad \delta \le 0 \end{aligned} \tag{10}$$

The first piece is the starting point for all exchange rate determination models, namely purchasing power parity (PPP), and simply states that exchange rate changes should reflect inflation rate differentials between the two countries. That is, for an exchange rate expressed in dollars per unit of foreign currency, when U.S. inflation is high the exchange rate increases and the dollar depreciates.

8 See, for example, Fama and Bliss (1987), recently updated by Fama (2004), for U.S. data and Jorion and Mishkin (1991) for international evidence.

9 It is straightforward to build expectations inconsistent with EHIR, such as $f_t^{1,2} = \alpha E_t[i_{t+1}] + (1-\alpha)E_t[\pi_{t+1,t+2}]$, in which the parameter α controls the extent to which there are violations of EHIR. If α = 1, EHIR holds. If α = 0, forward rates reflect fundamentals only (i.e., expected inflation), and do not anticipate any distortions associated with the Taylor rule. The theoretical model's capability to capture the empirical evidence carries through for $\alpha \neq 1$. In fact, imposing the expectations hypothesis reduces the degrees of freedom available in the model and thus potentially reduces its ability to match the empirical evidence.

The second piece reflects that, with the Taylor rule distortion given in equation (7), expected real rates are no longer equal across countries. Countries with expected inflation that is high relative to their mean target inflation levels will have higher expected real rates. What do differences in expected real rates then imply about exchange rate determination?

A popular description for exchange rate determination can be found in the literature on the "carry trade" in which investors borrow in low interest rate currencies and invest in high interest rate currencies. Specifically, a relatively high expected real rate in the U.S. causes capital inflows, dollar appreciation, and a fall in the exchange rate (e.g., Burnside, Eichenbaum, Kleschelski, and Rebelo (2006), Clarida, Davis, and Pedersen (2009), Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2009), Jorda and Taylor (2009), and Jurek (2009), among others). One preferred explanation is that the carry trade and resulting appreciation of the currency is compensation for the possibility of a crash in the currency's value – the so-called "up the stairs, down the elevator" description of high interest rate currencies (e.g., Brunnermeier, Nagel, and Pedersen (2009) and Plantin and Shin (2010)).

This view of the carry trade and crash risk premia has a theoretical basis in Farhi and Gabaix (2008), but can be viewed more generally in the context of the larger literature that argues for expected currency appreciation due to the existence of a time-varying risk premium that is negatively correlated with interest rate differentials (see, e.g., Fama (1984), Bekaert (1996), Mark and Wu (1998), Backus, Foresi, and Telmer (2001), Graveline (2006), Lustig and Verdelhan (2007), Lustig, Roussanov and Verdelhan (2008), Verdelhan (2010), and Backus, Gavazzoni, Telmer, and Zin (2010)). The literature is not completely sold on the risk premium argument. Alternative stories focus on justifications based on limited arbitrage or segmentation in the foreign exchange market (e.g., Froot and Thaler (1990), Froot and Ramadorai (2005), Stein (2009), and Jylha and Suominen (2010)).

In the above formulation, δ is negative because a high expected real rate in the U.S. represents compensation for crash risk, or implies capital inflows and dollar appreciation within limits to arbitrage. In particular, if $\delta = 0$, purchasing power parity holds. If $\delta < 0$, real rate differentials have a permanent effect on exchange rates, i.e., the deviation from PPP persists and accumulates each period.

While we do not take a view on the precise description of the carry trade phenomena, our reduced form model does include a third component of exchange rate determination based on the crash intuition. There is a substantial body of evidence that PPP holds in the long run and is therefore an important building block for exchange rates (see, e.g., Abuaf and Jorion (1990), Kim (1990), Rogoff (1996), Lothian and Taylor (1996), Taylor (2001, 2002), and Imbs, Mumtaz, Ravn, and Rey (2005)). Specifically, while the carry trade component allows exchange rates to deviate from PPP due to differentials in real rates, we posit a positive probability that exchange rates will revert to PPP. For simplicity, and in order to facilitate the calculation of closed form solutions, we model this reversion as a crash back to PPP in a single period, but one could easily envision extending the model to richer patterns of reversion. D_{t+1} is a dummy variable that takes on the value 1 when a crash occurs, 0 otherwise. When a crash occurs, all deviations from PPP since the last crash, which occurred $\tilde{W}$ periods ago, are reversed. Initially, we model exchange rates reverting to PPP with a fixed probability p each period:

$$D_{t+1} = \begin{cases} 1 & \text{with prob. } p \\ 0 & \text{with prob. } 1\text{-}p \end{cases} \tag{11}$$

so that $E[D_{t+1}] = p$ and $\Pr[\tilde{W} = n] = p(1-p)^{n-1}$. Later on, in subsection C of this section, we generalize the probability of a correction to be state (i.e., PPP deviation) dependent.

B. Implications for Uncovered Interest Rate Parity at Short and Long Horizons

The model for interest rates, inflation rates, and exchange rates described by equations (7)–(11) has implications for the typical forward premium regression given in equation (2) and our novel forward interest rate differential regression in equation (6).

Consider first the UIP regression, $\Delta s_{t,t+1} = \beta_0 (i_{t,1} - i^*_{t,1}) + u_{t+1}$. Appendix B of the paper shows:

$$\beta_0 = \frac{\text{cov}(\Delta s_{t,t+1}, i_{t,1} - i^*_{t,1})}{\text{var}(i_{t,1} - i^*_{t,1})} = \frac{\theta}{\theta+\gamma} + \frac{\delta\gamma\theta}{\theta+\gamma} - \frac{\delta\gamma p\left[\theta + \dfrac{1-p}{1-(1-p)\theta}\right]}{\theta+\gamma} \, . \tag{12}$$

The UIP regression coefficient is the sum of three terms.

The first term reflects the direct effect of inflation differentials on exchange rate changes via PPP. If interest rates fully reflect fundamentals and there are no Taylor rule distortions ($\gamma = 0$ in equation (7)), then $\beta_0 = 1$ and UIP holds exactly. If there are no Taylor rule distortions, then expected real rate differentials are zero, and there are no deviations of exchange rates from PPP, i.e., the second and third terms in equation (12) are zero. If there are Taylor rule distortions ($\gamma > 0$), but exchange rates still follow PPP ($\delta = 0$), then $0 < \beta_0 < 1$. In this case, even though expected real rates are different across countries, this divergence has no effect on exchange rates because $\delta = 0$, there are no PPP violations to reverse, and again the second and third terms in equation (12) are zero. For example, if inflation is persistent (e.g., $\theta = 0.8$ in equation (8)), and the typical Taylor rule adjusts interest rates by half the amount that inflation diverges from its target mean (e.g., $\gamma = 0.5$ in equation (7)), then the coefficient in the UIP regression is $\beta_0 = 0.62$. Even if PPP holds, then UIP can be violated as the Taylor rule implies that interest rates respond more to inflation shocks than exchange rates. Of course, a similar result would hold if the response was behavioral in nature (e.g., overreaction along the lines of Burnside, Han, and Hirshleifer (2011)).

The second term reflects the carry trade component of exchange rate changes due to expected real rate differentials. If there are Taylor rule distortions ($\gamma > 0$), and the real rate, carry trade component affects exchange rates ($\delta < 0$) so that PPP does not hold, then the second term in equation (12) is negative. If there are no crashes, then the third term is zero, and the regression coefficient is strictly less than that when PPP does hold. In fact, β_0 can now go negative, when $\delta\gamma < -1$, and the coefficient decreases as δ decreases. In other words, the carry trade effect works in achieving the correct coefficient provided in Table 1, Panel C. For example, under the parameters for inflation and the Taylor rule described above (i.e., $\theta = 0.8$, $\gamma = 0.5$), for δ equal to -1, -5, and -10, β_0 equals 0.31, -0.92, and -2.46, respectively. From an economic standpoint, a value of δ = -5 implies that a domestic currency will appreciate 5% over the following year if domestic real rates are expected to be 1% higher than those in the foreign country.

The final term characterizes the reversion component. If there are Taylor rule distortions ($\gamma > 0$), a carry trade component ($\delta < 0$), and a positive probability that exchange rates will revert back to PPP, then the effect of the crash component on the regression coefficient partially (or even fully) reverses the effect of the carry trade. In fact, if the probability of a crash back to PPP is one every period, then the carry trade effect disappears and $\beta_0 = \frac{\theta}{\theta+\gamma}$. The carry trade effect on exchange rates is reversed by an immediate reversion back to PPP, and there are no longer any PPP violations. The intuition behind the third term in equation (12) is that the current interest rate differential has information not only about future real rate differentials, which, due to the carry trade, leads to exchange rate appreciation, but also about past interest rate differentials. Depending on the probability of a crash, these past differentials tell us something about the future magnitude of the crash in the exchange rate. For example, under the parameters for inflation and the Taylor rule described above (i.e., $\theta = 0.8$, $\gamma = 0.5$), and considering δ = -5 and -10, β_0 increases from -0.92 to -0.33, and from -2.46 to -1.27, respectively as the probability of a crash (p) goes from 0% to 7%.

The forward premium regressions expressed in terms of longer maturity forward interest rate differentials in equation (6) produce very different results than the standard UIP regressions in equation (2). Therefore, we next turn to the implications of our reduced form model given in equations (7)-(11) for the UIP regression with forward interest rate differentials, $\Delta s_{t,t+1} = \beta_1 (if_{t-1}^{1,2} - if_{t-1}^{1,2^*}) + u_{t+1}$.

Appendix B of the paper shows:

$$\beta_1 = \frac{\operatorname{cov}(\Delta s_{t,t+1}, if_{t-1}^{1,2} - if_{t-1}^{1,2^*})}{\operatorname{var}(if_{t-1}^{1,2} - if_{t-1}^{1,2^*})} = \frac{\theta}{\theta+\gamma} + \frac{\delta\gamma\theta}{\theta+\gamma} - \frac{\delta\gamma p\left[\theta + \frac{1-p}{\theta}\left(\theta + \frac{1-p}{1-(1-p)\theta}\right)\right]}{\theta+\gamma} \quad . \tag{13}$$

The first two terms are identical to those of β_0 in equation (12). Under the expectations hypothesis of interest rates, which we impose in our model, the forward interest rate differential is the expected future spot interest rate differential, and regressing exchange rate changes on these two quantities yields identical results in a world without reversion to PPP.

The difference between the two coefficients is

$$\beta_1 - \beta_0 = \frac{\delta\gamma p(1-p)^2\left[\dfrac{\theta^2 - 1}{(1-(1-p)\theta)\theta}\right]}{\theta + \gamma}. \tag{14}$$

If there is either a 0% or 100% probability of a crash, then the two UIP coefficients are the same. Interestingly, and consistent with the regression results in Table 2, for economically relevant parameter values β_1 is always greater than β_0 when the crash probability $0 < p < 1$. For example, under the parameters for inflation and the Taylor rule described above (i.e., $\theta = 0.8$, $\gamma = 0.5$), and a crash probability of $p = 7\%$, and considering $\delta = -5$ and -10, $\beta_0 = -0.33$ versus $\beta_1 = -0.12$, and $\beta_0 = -1.27$ versus $\beta_1 = -0.86$, respectively.

The intuition is that the forward interest rate differential has more information about the magnitude of the existing deviation from PPP, and thus the impact of a currency crash, than does the current interest rate differential. The current interest rate differential contains information about prior inflation rate differentials, and thus the buildup of PPP violations, due to the persistence of inflation, but this information decays as one goes back in time. In contrast, the lagged forward interest rate differential captures the actual inflation differential last period. This same differential has information both about the current inflation differential and inflation differentials further back in time, again due to the persistence of inflation. This intuition also suggests that if inflation is persistent, so that Taylor rule deviations will persist, then long-horizon forward interest rate differentials (even if stale) will contain considerable information about the magnitude of future currency crashes. Thus, the difference in equation (14) may increase with horizon, a main finding from the regression results in Table 2.

We could calculate the forward interest rate differential coefficients for longer horizons in closed form (similar to equation (13)), but the horizon dependence is most easily illustrated using numerical results for reasonable parameter values. Therefore, we simulate the model given in equations (7)–(11), and report the regression results in Table 3. Table 3 provides results for the standard UIP regression with interest rate differentials and for the forward interest rate differential regressions in equation (6), for horizons of 1-4 years, for a variety of parameters: $\gamma = 0.0$, 0.3, 0.5, and 0.7; $\delta = 0$, -5, -10, -15, and -20; $\theta = 0.8$; and $p = 7\%$. Shocks to inflation in the two countries, as given in equation (8), are assumed to be normally distributed, and the coefficient estimates are not affected by the choice of the variance of these innovations or their correlation across countries.

The top panel illustrates the point made above that, when there are no Taylor rule distortions (γ = 0), the coefficient equals 1 at all horizons. The small deviations from one in the second decimal place indicate the precision of the simulated coefficient estimates relative to their true values. The top line in each of the four panels illustrates the second point that, when there is no carry trade effect (δ = 0), the coefficient is independent of the horizon. However, this coefficient declines as the magnitude of the Taylor rule distortion increases because interest rates are a magnified function of inflation.

Most important, for each non-zero value of γ (i.e., Taylor rule distortion) and δ (i.e., carry trade effect), the coefficient is increasing in the horizon. The UIP regression coefficient (β_0) and the rate of increase depend jointly on the two parameters. Holding the magnitude of the Taylor rule distortion fixed, increasing the magnitude of the carry trade effect (moving down the lines within a panel), decreases the coefficients at the short horizon, as argued above, and for sufficiently large magnitudes the UIP regression coefficient is negative. The carry trade parameter is also the primary determinant of the range of the coefficients from short to long horizons, with this range increasing in the magnitude of δ. However, there is also clearly an interaction effect between the Taylor rule distortion and the carry trade effect.

Reasonable parameterizations (e.g., γ = 0.5, δ = -5) can induce a switch in the sign of the coefficient as the horizon increases. For short horizons, the carry trade effect dominates and the coefficient is negative. For longer horizons, the role of the forward interest rate differential as a proxy for the magnitude of the PPP violation and hence the size of a crash, should it occur, becomes the more important factor, and the coefficient becomes positive. However, the magnitudes of the coefficients at longer horizons generated by the model in this scenario are smaller than those in the data.

Finally, the last 5 columns of Table 3 present the R^2s from the regressions. As expected, when there is no carry trade component, the R^2s are high and decrease in the horizon of the regression. Exchange rate changes depend on realized inflation differentials, which are predicted well by spot interest rate differentials, but less so by lagged forward interest rate differentials. The R^2s are much lower when there is a carry trade component and crashes back to PPP that reverse this component. The magnitudes of the R^2s are of less interest because they depend critically on the fact that we assume reversion to PPP occurs in a single period, creating large exchange rate moves that dominate the variation in exchange rate changes. More interesting are the patterns in these R^2s. For large carry trade effects, the coefficient in the standard UIP regression is large in magnitude (e.g., -2.89 for γ = 0.7, δ = -15 versus -0.32 for γ = 0.5, δ = -5), and the relative R^2 is also large (2.36% versus 0.41%). This explained variation declines in horizon as the coefficient begins to pick up the offsetting crash component. For example, for γ = 0.7, δ = -15, the R^2s are 2.36%, 0.83%, 0.17%, and 0.00% for horizons 0 to 3. However, as the crash component begins to dominate at even longer horizons, the R^2 can increase, e.g., from 0.00% to 0.03% for the parameters above.

C. Extending the Exchange Rate Model

The reduced form model described in Section III.A above can potentially be extended in several ways to better fit existing stylized facts. One natural generalization is to relax the assumption in the exchange rate determination model described in equation (10) of a constant probability of a currency crash back to PPP. The purpose of this assumption is to allow for closed-form expressions for the coefficients in the exchange rate regressions in equations (2) and (6). However, both theories based on speculative dynamics (e.g., Plantin and Shin (2010)) and existing empirical work (e.g., Brunnermeier, Nagel, and Pedersen (2009) and Jorda and Taylor (2009)) imply that this probability should be increasing in the deviation from PPP. In other words, as the exchange rate moves further and further from its fundamental PPP relation, the tension to bring it back increases.

We model the time-varying crash probability in a simple way:

$$p_t = \frac{w\left|PPPD_t\right|}{1+w\left|PPPD_t\right|}, \tag{15}$$

where $|PPPD_t|$ is the absolute deviation of the exchange rate from its value implied under PPP at time t, and w is a scalar chosen to match a specific unconditional crash probability, which we denote $\overline{p}$. Thus, p_t varies through time, increasing in the current deviation of the exchange rate from PPP.

Table 4, Panel A presents what is essentially a rough calibration of this extended model to the empirical results in Table 2. We present both the slope coefficients for the forward premium regression for horizons up to 4 years and the associated R^2s. In addition to varying the magnitude of the Taylor rule distortion (γ) and the carry trade effect (δ), we also vary the persistence of the inflation process (θ) and the unconditional crash probability ($\overline{p}$), which amounts to varying the parameter w in equation (15). We consider variations in the parameters around a plausible benchmark of $\gamma = 0.5$, $\delta = -10$, $\overline{p} = 7\%$, and $\theta = 0.8$.

The results for the benchmark parameterization are presented in the first row of the table, and it is clear why we have chosen these parameter values. The coefficient in the UIP regression is negative, it switches signs for the regression with forward interest rate differentials at a horizon of 1 year, and it increases in horizon to a value substantially greater than one at longer horizons, all consistent with the empirical evidence presented in Table 2. In other words, our reduced form model of exchange rates is able to explain the striking results presented earlier.

The intuition for these results is the same as that discussed earlier for the simpler model with a constant crash probability. The interest rate differential picks up the carry trade effect, which reverses the sign of the coefficient relative to the standard UIP intuition. However, there is a second offsetting effect.

Spot and forward interest rate differentials also proxy for the magnitude of the deviation of exchange rates from PPP. This deviation will be reversed at some point, and this reversal is, by definition, a movement of exchange rates in the direction opposite to the carry trade effect. At the short horizon, the former effect dominates. At a horizon of 1 year, the effects are almost offsetting, and the coefficient is close to zero. However, at long horizons the crash effect becomes more important. Because crashes are relatively rare, large deviations from PPP can build up, and the resulting exchange rate move will be large, thus the coefficient can exceed one at long horizons.

The patterns in the regression R^2s are consistent with the empirical results of Table 2. At short horizons, explained variation is low because the independent variable is picking up both the carry trade effect and the offsetting reversion to PPP. However, at long horizons, the crash effect is dominant and the R^2 is many times larger than at the shortest horizon, i.e., 0.21% at horizon 0 versus 1.41% at horizon 4.

The subsequent pairs of parameterizations below the benchmark case in Table 4 illustrate the marginal effects associated with each parameter in the model. For each pair, we perturb a single parameter, highlighted in bold, above and below its level in the benchmark case. The marginal effects of the Taylor rule distortion (γ) and the carry trade (δ) are similar. In both cases, as these parameters increase in magnitude, the horizon effect increases, i.e., the short-horizon coefficient becomes more negative, and the long-horizon coefficient becomes more positive. This magnification of the horizon effect occurs because both the PPP violations, via the carry trade effect, and the size of the associated crashes back to PPP increase as the magnitudes of γ and δ increase. In the former case, for a given inflation differential, the magnitude of the interest rate differential and the corresponding expected real rate differential is larger, while in the later case, a given expected real rate differential has a larger effect on exchange rates.

Holding the other parameters constant, decreasing the persistence of inflation also causes a magnification of the horizon effect. This decrease reduces the relation between forward interest rates and spot interest rates, i.e., the expectation hypothesis of interest rates still holds, but the innovation in these expectations over time is relatively larger and inflation reverts more quickly to its mean. As a result, spot interest rate differentials continue to contain information about the carry trade effect but contain less information about lagged interest rate differentials and thus the magnitude of existing PPP violations. Similarly, forward interest rate differentials contain information about the build-up of PPP violations in the corresponding period, but they contain less information about future spot interest rates and thus the future carry trade effect. This improved separation of the two effects increases the coefficient estimates.

Finally, the unconditional crash probability shifts the coefficients at all horizons in the same direction. As the crash probability decreases, the coefficients decrease as well. As a crash becomes less likely in any given year, the carry trade effect, which generates a negative relation between interest rate differentials and exchange rates, becomes more important relative to the crash effect.

Of course, while the model is relatively simple, the relations between the exchange rates and interest rate differentials are nonlinear, and the above analysis does not capture all the potential complex interactions. Moreover, the marginal effect of a single parameter does depend on the values of the other variables.

D. Additional Implications of the Model

Sections III.B and III.C above show that the forward premium regressions of equations (2) and (6) have considerable common information. In the closed-form solutions, two of the three terms in the regression coefficients are identical, and the third one has a similar structure. The fact that the β_0 and β_1 regression coefficients in equations (12) and (13) are similar should not be surprising. Under the expectations hypothesis of interest rates:

$$\begin{aligned} & if_{t-1}^{1,2} - if_{t-1}^{1,2^*} = E_{t-1}\left(i_{t,1} - i_{t,1}^*\right) \\ & \Rightarrow i_{t,1} - i_{t,1}^* = if_{t-1}^{1,2} - if_{t-1}^{1,2^*} + \eta_t^{\Delta if} \end{aligned}, \qquad (16)$$

where $\eta_t^{\Delta if}$ is the forecast error associated with forward interest rate differentials with the property that $\text{cov}\left(if_{t-1}^{1,2} - if_{t-1}^{1,2^*}, \eta_t^{\Delta if}\right) = 0$. What is surprising is that these two closely related independent variables can generate such different regression coefficients, especially at longer horizons, both in the data and in our simple model. As argued above, the explanation is that both variables capture two offsetting effects. As the balance between these effects changes at different horizons, the sign of the coefficient also changes.

Given the apparent existence of two offsetting effects, a logical step would be to attempt to disentangle these effects in a bivariate regression. In the context of the model, any two variables, e.g., the spot and forward interest rate differentials, will do as long as they are not perfectly correlated. From an empirical perspective, we need to find variables that are less correlated in order to avoid multicollinearity problems in our relatively small sample. A natural approach is to separate the interest rate differential into two terms, its expected value based on the forward interest rate differential $(if_{t-1}^{1,2} - if_{t-1}^{1,2^*})$, and the unexpected shock to interest rates over this period, as measured by $(i_{t,1} - i_{t,1}^*) - (if_{t-1}^{1,2} - if_{t-1}^{1,2^*})$.[10] In particular, consider the following regression:

$$\Delta s_{t,t+1} = \phi_0[(i_{t,1} - i_{t,1}^*) - (if_{t-1}^{1,2} - if_{t-1}^{1,2^*})] + \phi_1(if_{t-1}^{1,2} - if_{t-1}^{1,2^*}) + u_{t+1} \ . \qquad (17)$$

[10] This separation would strictly be true only under EHIR. Footnote 9 discusses deviations from EHIR.

Under EHIR, these variables are uncorrelated. Under UIP both coefficients will equal one. However, under the reduced form model of equations (7)–(11), with a constant crash probability, Appendix B shows that

$$\phi_0 = \frac{\theta}{\theta+\gamma} + \frac{\delta\gamma\theta}{\theta+\gamma} - \frac{\delta\gamma p(\theta+1-p)}{\theta+\gamma}$$
$$\phi_1 = \beta_1 = \frac{\theta}{\theta+\gamma} + \frac{\delta\gamma\theta}{\theta+\gamma} - \frac{\delta\gamma p\left[\theta + \frac{1-p}{\theta}\left(\theta + \frac{1-p}{1-(1-p)\theta}\right)\right]}{\theta+\gamma} \quad . \tag{18}$$

The coefficient ϕ_1 equals β_1 from equation (13) because the independent variables are uncorrelated. ϕ_0 is a slightly simplified version of β_0, and, for $0 < p < 1$, $\phi_1 > \phi_0$. The difference between the coefficients is

$$\phi_1 - \phi_0 = -\frac{\delta\gamma p \frac{(1-p)^2}{(1-(1-p)\theta)\theta}}{\theta+\gamma}. \tag{19}$$

Of particular interest, it is possible for ϕ_1 to be positive and ϕ_0 to be negative. For example, under reasonable parametervalues for inflation and the Taylor rule (θ = 0.8, γ = 0.3, δ = -5 and p=7%), we get ϕ_1=0.20 and ϕ_0= -0.20. Because $(i_{t,1} - i^*_{t,1})$ is broken into $(if^{1,2}_{t-1} - if^{1,2^*}_{t-1})$ and $(i_{t,1} - i^*_{t,1}) - (if^{1,2}_{t-1} - if^{1,2^*}_{t-1})$, the opposite coefficients for ϕ_1 and ϕ_0 mean, in practice, that the standard UIP regression coefficient, β_0, will be close to zero and generate low R^2s, the typical finding in this literature for exchange rate determination. For example, under the above parameter values, β_0=0.06.

Table 4, Panel B provides simulation results for multiple horizons for the extended model with time-varying crash probabilities for the regression

$$\Delta s_{t,t+1} = \alpha + \phi_{0,j}[(i_{t,1} - i^*_{t,1}) - (if^{j,j+1}_{t-j} - if^{j,j+1^*}_{t-j})] + \phi_j(if^{j,j+1}_{t-j} - if^{j,j+1^*}_{t-j}) + u_{t+1}. \tag{20}$$

The second subscript on the ϕ_0 coefficient indicates that the variable is the difference between the spot interest rate differential (at horizon 0) and the forward interest rate differential at horizon j. We report results for the same parameter values used in Table 4.

As expected, ϕ_j, i.e., the coefficient on the forward interest rate differential, equals β_j, i.e., the coefficient on the forward interest rate differential in the univariate regression in Table 4, Panel A up to some small amount of simulation noise. More interesting are the coefficients on the innovation term ($\phi_{0,j}$). These coefficients are significantly more negative than the coefficient in the UIP regression (β_0

in Table 4, Panel A) because the second variable is now controlling for the crash effect that was attenuating the carry trade effect primarily picked up by the spot interest rate differential. In all cases, the magnitudes decrease slightly in horizon with changes in the balance between the offsetting effects captured by the two variables.

The R^2s of the regressions are also interesting. They exceed those of the univariate forward premium regression by a significant amount at all horizons, e.g., from 1.23% to 2.50% versus 0.06% to 1.41% at horizons 1 to 4 for the benchmark parameterization. Moreover, they are increasing in horizon because spot and forward interest rate differentials that are separated further in time provide more information about the offsetting carry trade and crash effects.

While the regression in equation (20) clearly provides a useful decomposition of the carry trade and crash effects, it is a somewhat indirect approach to this problem. In the model, there are two key state variables—the interest rate differential and the deviation of the current exchange rate from that implied by purchasing power parity. The former variable picks up the carry trade effect, while the latter measures the size and direction of the exchange rate move in the event of a reversion (crash) to fundamentals (PPP) and also the probability of such a reversion in the extended model with a time-varying crash probability. Consequently, a natural analysis is a regression of exchange rate changes on these two variables, i.e.,

$$\Delta s_{t,t+1} = \alpha + \psi_1 PPPD_t + \psi_2 (i_{t,1} - i^*_{t,1}) + \varepsilon_{t+1}. \tag{21}$$

Table 4, Panel C provides simulation results for this regression and special cases thereof.

For the benchmark parameter values, we report results for the standard UIP regression, which are also reported in the first line of Table 4, Panel A; for the regression with only the PPP deviation variable; and for the bivariate regression. When include alone, the PPP deviation has a negative coefficient, i.e., deviations will be reversed in the future, and the R^2 is high relative to the regressions with interest rate differentials, i.e., 7.75% versus a maximum of 1.41% in Panel A and 2.50% in Panel B. The actual deviation from PPP is a better predictor than interest rate differentials that provide a noisy proxy for this deviation based on the inflation differential in a single period. When included together, the magnitude of the coefficients on both variables increase dramatically from their counterparts in the univariate regressions—from -0.13 to -0.21 on the PPP deviation and from -0.41 to -2.67 on the interest rate differential. While the deviation from PPP measures the magnitude of a crash, should it occur, it is also related to current inflation differentials and hence the carry trade component as well. Including a direct measure of this component thus increases the explanatory power of both variables.

IV. Empirical Analysis of the Exchange Rate Model

The theoretical results of Section III provide a way to reconcile the forward premium anomaly (regression equation (2)) with the forward interest rate differential results (regression equation (6)) across multiple horizons. This section analyzes the additional implications of the model described in Section III.D.

Table 5 presents results for the regression

$$\Delta s_{t,t+1} = \alpha_j + \phi_{0,j}[(i_{t,1} - i_{t,1}^*) - (if_{t-j}^{j,j+1} - if_{t-j}^{j,j+1^*})] + \phi_j(if_{t-j}^{j,j+1} - if_{t-j}^{j,j+1^*}) + u_{t+1}. \tag{22}$$

for each exchange rate and horizon. Across all of the horizons and across both currencies, the coefficient ϕ_j is always positive, and generally increasing (albeit noisily) in horizon, consistent with the simulation results in Table 4, Panel B. In contrast, the $\phi_{0,j}$ coefficients are all negative and declining in magnitude as the horizon increases, again in line with the simulation evidence. The R^2s are quite impressive.

For example, for the USD/DEM exchange rate, at forward rate horizons of one to four years, the ϕ_js are 0.53, 0.24, 1.32, and 2.39, respectively, while the $\phi_{0,j}$s are –2.28, –1.01, –0.96, and –0.89. Similar to the results in Table 2, it is difficult to reject the hypothesis that the coefficients on the forward interest rate differential are equal to 1. In Panel B, only the Wald test across horizons and currencies is able to reject this hypothesis at the 5% level, with an additional rejection at the 10% level for the Wald test across horizons for the USD/GBP. In contrast, there is convincing statistical evidence that the coefficient on the unexpected component is not equal to 1. In Panel C, a large number of both the Wald and LM statistics are significant at the 5% level.

The negative $\phi_{0,j}$ explains why the forward premium anomaly exists from a statistical viewpoint—that is, why we get negative coefficients and low R^2s in Table 1, Panel C. Breaking up current interest rates into the two components separates information about the magnitude and probability of future currency crashes contained in the forward curve from current interest rates. Including them together, the two information sources offset each other, leading to a low R^2.

From the standpoint of the model, the positive and increasing coefficients on the forward interest rate differentials are capturing the probability and magnitude of a currency crash back to PPP, while the negative coefficients on the forecast error in the exchange rate regression are capturing the carry trade effect.

We argue that dated (i.e., old) information in forward rate differentials is important because these differentials predict the crash component of future changes in exchange rates. An alternative story is that there is simply additional information in the term structure of interest rates about future exchange rates that has nothing to do with the fundamentals of our model. If this were the case, then presumably forward interest rates contemporaneous with the spot rates in the standard forward premium regression would provide better information than lagged differentials, since they contain newer information. In the context of the model in Section III, adding contemporaneous forward interest rate differentials would have no effect at all on the explanatory power. They contain identical information to that in the spot interest rate differentials about inflation rate differentials.

Table 6 investigates this conjecture via estimation of the regressions

$$\Delta s_{t,t+1} = \alpha_j + \phi_{0,j}[(i_{t,1} - i^*_{t,1}) - (if_t^{\,j,j+1} - if_t^{\,j,j+1^*})] + \phi_j(if_t^{\,j,j+1} - if_t^{\,j,j+1^*}) + u_{t+1}\,. \tag{23}$$

These regressions, with results presented in Table 5, Panel A, are analogous to those in equation (22) except that the regressor is the contemporaneous (time *t*) forward interest rate instead of the lagged forward interest rate. The notable result is that the R^2s of these regressions are extremely low, ranging from 1.87% to 2.51%, versus a range of 3.74% to 16.92% for the regressions using lagged forward interest rate differentials in Table 5, Panel A. Using contemporaneous forward interest rate differentials, the predictive power is similar to the predictive power when using the current spot interest rate differential alone (see Table 1, Panel C and Table 2, Panel A). These results confirm the validity of the exchange rate model developed above in Section III. The innovation in current forward interest rates relative to lagged rates reduces their power to explain exchange rate changes.

Next we turn to the bivariate regression using the deviation of the exchange rate from PPP. This deviation is unobservable, but we can construct a variable that captures the same information, up to a constant. Specifically, consider the log real exchange rate

$$q_t = s_t + (z^*_t - z_t)\,, \tag{24}$$

where q and s are the log real and nominal exchange rates, respectively, and z and z^* denote the log price levels in the domestic and foreign country, respectively. Under PPP, the real exchange rate is constant; thus, the observed real exchange rate equals the deviation of this exchange rate from this PPP implied level, up to an unknown constant. In the context of a regression analysis, this unknown constant will appear in the intercept.

Table 7, Panel A presents summary statistics for the log real exchange rate series for the two currency pairs. The means are essentially meaningless in that they reflect the normalization of the price level series in the two countries. It is not surprising that the two series are very persistent given the persistence of the exchange rate series, and the relatively strong positive correlation between the two series is also expected.

We estimate regressions of annual exchange rate changes (overlapping monthly) on the log real exchange rate and the interest rate differential at the beginning of the year (and special cases thereof):

$$\Delta s_{t,t+1} = \alpha + \psi_1 q_t + \psi_2(i_{t,1} - i^*_{t,1}) + \varepsilon_{t,1}\,. \tag{25}$$

The results are reported in Table 7, Panel B. For ease of comparison, the top line for each exchange rate reports the standard UIP regressions, which are also reported in Tables 1 and 2. The second line reports the regression with the log real exchange rate, and the final line reports the results from the full specification.

The specification in equation (25) is essentially the same as that estimated in Jorda and Taylor (2009). They motivate the real exchange rate variable as the deviation from the fundamental equilibrium exchange rate, although they do not provide a motivating model since they are primarily interested in forecasting and the associated trading strategies. They estimate various models using monthly data across multiple exchange rates for the period 1986-2008 and report results consistent with ours.

The first notable result in Table 7, Panel B is that, both alone and in the full specification, the log real exchange rate appears with a negative and statistically significant coefficient. This negative coefficient is consistent with the intuition from the model. When the real exchange rate is high, i.e., the dollar has appreciated less or depreciated more than would be suggested by the relative inflation rates in the two countries, then this effect is expected to reverse in the coming year. Moreover, this reversion to PPP, or expected currency crash, explains a significant fraction of the variation in exchange rate changes on its own, with R^2s of 18.8% and 17.5% for the USD/GBP and USD/DEM, respectively.

The second notable result is that including the interest rate differential increases both the magnitude of the coefficient on this variable, relative to the case when it is used on its own, and the explanatory power of the regression. For example, the coefficient for the USD/DEM goes from -0.72 in the standard UIP regression to -1.70 in the augmented regression, and the corresponding R^2 increases to 29.4% (from 1.8% in the UIP regression and 17.5% in the real exchange rate regression). Clearly, controlling for both the crash effect and the carry trade effect together enhances our ability to identify both effects and increases the explanatory power for exchange rates, consistent with our model.

V. Concluding Remarks

The forward premium puzzle is one of the more robust and widely studied phenomena in financial economics. Our paper makes three important contributions to this large literature.

First, we document that recasting the UIP regression in terms of forward interest rate differentials, rather than spot interest rate differentials, deepens the puzzle. Specifically, the coefficients in these regressions are positive in contrast to the negative coefficients in the standard UIP specification, and the R^2s are generally increasing in the horizon.

Second, we present a model that can both explain the existing evidence and reconcile it with our new evidence. The key insight of the model is that exchange rate changes reflect two distinct but related phenomena. A carry trade effect associated with real rate distortions pushes exchange rates in the opposite direction to that predicted by a standard model of PPP. However, exchange rates probabilistically crash back to their fundamental levels. Forward interest rate differentials at different horizons pick up both of these conflicting effects to different degrees, yielding horizon-dependent coefficients and R^2s.

Finally, we show that within the model it is possible to decompose these two effects, either using forward rate differentials and shocks to these differentials, or interest rate differentials and real exchange rates. The data are consistent with these theoretical decompositions and provided further support for our model of exchange rate determination.

While we present the simplest model that is broadly consistent with the empirical evidence, the model can be generalized across a number of dimensions. For example, we could add a real side to the economy, we could make the Taylor rule more complex, we could incorporate violations of the expectations hypothesis of interest rates, and we could postulate different dynamics for reversion to PPP. All of these adaptations could also be asymmetric, i.e., they could look different in the two countries. We believe that models along these lines could potentially explain much of the richness in the data.

Appendix A: Monte Carlo Exercise

One potential concern with the results reported in Table 2 is that the standard errors are spuriously low and the R^2s are spuriously high due to small sample problems in the regressions. We argue in Section II.C that the overlap problem is not that serious due to the relatively low predictability of exchange rate changes, but it is still important to verify this conjecture. Consequently, we construct a Monte Carlo experiment in which we employ a VAR for the relevant forward interest rate differentials, spot interest rate differentials, and changes in exchange rates, imposing the expectations hypotheses of interest rates and using two different models for exchange rates. In one experiment we impose the expectations hypothesis for exchange rates, i.e., we assume uncovered interest rate parity holds, and in the other experiment we assume exchange rates follow a random walk, i.e., exchange rate changes are unpredictable.

We also consider two different distributional assumptions for the shocks to exchange rate changes. In the first analysis, we assume that the shocks across all equations follow a multivariate normal distribution. In the second analysis, we resample the shocks to exchange rates from the series of monthly exchange rate changes observed in the data. We then simulate these models, generating 100,000 replications of 428 monthly observations. For each replication, we aggregate the data to an annual frequency, as in the empirical analysis, and we then estimate equation (6). For comparison purposes, we also estimate the long-horizon regression version of equation (2) following Chinn and Meredith (2005). Thus we can assess the small sample properties of our specification and also compare them to those of the alternative long-horizon regressions.

Specifically, for the first experiment, we assume that the expectations hypotheses of exchange rates and interest rates hold at a monthly frequency, and that the longest maturity forward rate differential (the forward rate from month 59 to month 60) follows an AR(1) process:[11]

[11] Throughout this appendix, periods are measured in months (in contrast to the rest of the paper where all periods are measured in years).

$$
\begin{aligned}
&\Delta s_{t,t+1} = i_{t,1} - i_{t,1}^{*} + \varepsilon_{t,t+1}^{s} \\
&i_{t+1,1} - i_{t+1,1}^{*} = if_t^{1,2} - if_t^{1,2^*} + \varepsilon_{t,t+1}^{1} \\
&if_{t+1}^{1,2} - if_{t+1}^{1,2^*} = if_t^{2,3} - if_t^{2,3^*} + \varepsilon_{t,t+1}^{2} \\
&if_{t+1}^{2,3} - if_{t+1}^{2,3^*} = if_t^{3,4} - if_t^{3,4^*} + \varepsilon_{t,t+1}^{3} \\
&\vdots \\
&if_{t+1}^{58,59} - if_{t+1}^{58,59^*} = if_t^{59,60} - if_t^{59,60^*} + \varepsilon_{t,t+1}^{59} \\
&if_{t+1}^{59,60} - if_{t+1}^{59,60^*} = \rho(if_t^{59,60} - if_t^{59,60^*}) + \varepsilon_{t,t+1}^{60}
\end{aligned}
\tag{26}
$$

where

$$
\varepsilon_{t,t+1} \equiv \begin{bmatrix} \varepsilon_{t,t+1}^{s} \\ \varepsilon_{t,t+1}^{1} \\ \varepsilon_{t,t+1}^{2} \\ \vdots \\ \varepsilon_{t,t+1}^{59} \\ \varepsilon_{t,t+1}^{60} \end{bmatrix} \sim MVN(0, \Sigma)
\tag{27}
$$

We impose the following structure on the covariance matrix of the shocks:

$$
\Sigma = \left[\begin{array}{c|ccccc}
\sigma_s^2 & 0 & \cdots & 0 & \cdots & 0 \\
\hline
 & \sigma_i^2 & \cdots & \upsilon_{ij}^{t-1}\rho_{ij}\sqrt{\upsilon_i^{t-1}}\sigma_i^2 & \cdots & \upsilon_{ij}^{59}\rho_{ij}\sqrt{\upsilon_i^{59}}\sigma_i^2 \\
 & & \ddots & & & \vdots \\
 & & & \upsilon_i^{t-1}\sigma_i^2 & & \upsilon_{ij}^{59-t}\rho_{ij}\sqrt{\upsilon_i^{t+58}}\sigma_i^2 \\
 & & & & \ddots & \vdots \\
 & & & & & \upsilon_i^{59}\sigma_i^2
\end{array}\right]
\tag{28}
$$

Specifically, we impose that the variance of the shocks to forward interest rate differentials decline in maturity and that the correlations between the shocks to forward interest rate differentials decline in the difference between the maturities, at fixed rates determined by the parameters υ_i and υ_{ij}, respectively. We also impose zero correlation between the shock to exchange rate changes and the shocks to forward interest rate differentials. In the data, these correlations are relatively small and negative. However, these negative correlations are another manifestation of the violations of UIP that result in negative coefficients in the forward premium regressions in Tables 1 and 2. Therefore, we set the correlations to zero for the purposes of the Monte Carlo analyses.

We calibrate the parameters of the model in order to match approximately the covariance matrix of the annual exchange rate changes and the annual spot and forward interest rate differentials, and the autocorrelation of the 4- to 5-year forward interest rate differentials. Obviously, these values differ somewhat across the two exchange rates we employ in the empirical analysis, so we target intermediate values. The inferences drawn from the Monte Carlo analysis are not sensitive to the precise choice of the parameters.

Define the state vector

$$y_{t+1} = \begin{bmatrix} \Delta s_{t,t+1} \\ i_{t+1,1} - i^{*}_{t+1,1} \\ if^{1,2}_{t+1} - if^{1,2^{*}}_{t+1} \\ \vdots \\ if^{58,59}_{t+1} - if^{58,59^{*}}_{t+1} \\ if^{59,60}_{t+1} - if^{59,60^{*}}_{t+1} \end{bmatrix} \tag{29}$$

Equations (26)-(27) imply that $y_{t+1} \sim MVN(0,\Omega)$, where Ω is a function of ρ and Σ. The simulation procedure is as follows:

1. Draw starting values y_t from the distribution $y_t \sim MVN(0,\Omega)$.
2. Draw an error vector ε_{t+1} from the distribution $\varepsilon_{t+1} \sim MVN(0,\Sigma)$.
3. Compute y_{t+1} using this error vector and the lagged state vector via equation (26).
4. Return to step 2 above.

We generate 100,000 simulations of 428 monthly observations. We aggregate these 428 monthly data to an annual frequency and construct simulated samples with the appropriate lag structure of annual, monthly overlapping data of 357 observations each, the length of our sample. For each sample, we estimate the forward premium regressions in equation (6) and compute various test statistics. We also estimate the long-horizon versions of the forward premium regression in equation (2), after Chinn and Meredith (2005).

We also conduct a second Monte Carlo exercise, which is identical to the first except that we assume that exchange rates follow a random walk:

$$\Delta s_{t,t+1} = \varepsilon^{s}_{t,t+1} \ . \tag{30}$$

Finally, we repeat the analyses above, relaxing the restriction that the shocks to exchange rate changes are normally distributed in order to incorporate the possible effects of fat tails in the relevant distribution. Instead, we resample with replacement actual monthly exchange rate changes from either the USD/GBP or USD/DEM series. To preserve the excess kurtosis, but to eliminate any sample-specific mean or skewness effects, we augment the two series with an equal number of observations that correspond to the negative of the observed exchange rate changes.

The second and third to last columns of Table 2, Panel A, discussed in Section II.C, and Table A.1 report the key results. Table A.1, Panel A compares the R^2s from the regressions in equation (6), i.e., using forward interest rate differentials, to those from the long-horizon versions of the regression in equation (2), i.e., using long-horizon spot rate differentials, under the expectation hypothesis of exchange rates (β_j = 1). The statistics in this panel, and in the remainder of Table A.1, are calculated from simulations that resample from the USD/GBP exchange rate changes because this series exhibits the most excess kurtosis, but inferences from simulations under normality or using the USD/DEM exchange rate changes are similar. When one uses equation (6), the biases in the R^2s are clearly less severe than in the corresponding long-horizon regressions. As the horizon goes from one to four years, the bias, i.e., the difference between the mean R^2 from the simulations and the true R^2, ranges from 2.68% (5.92% simulated versus 3.24% true infinite sample R^2) to 2.60% for regressions using forward interest rates, versus an increase from 4.49% (11.28% simulated versus 6.79% true) to 6.71% for the long-horizon spot rate regressions.

Equally problematic for the long-horizon regressions, there is much less independent information in these regressions compared with the forward interest rate regressions. The correlations between the coefficient estimators range from 0.68 to 0.97 across the various horizons in the long-horizon regressions, in contrast to a much lower range of correlations, from 0.36 to 0.86, in the forward interest rate regressions.[12]

Table A.1, Panel B reports the results under the assumption that the exchange rate follows a random walk (β_j = 0). Again the forward interest rate regressions have smaller biases in R^2s relative to the long-horizon regressions, and there is considerably more independent information in the former regression system. The regressions using the forward interest rate differentials have a bias that ranges from 2.79% to 3.10%, while the biases in the long-horizon regressions increase with the horizon up to 9.99%. Overall, these simulation results suggest that small sample bias cannot explain the large differences in R^2s across horizons found in the data, and that the forward interest rate regressions have better statistical properties than the corresponding long-horizon regressions.

Table A.1, Panel C presents simulation results for the Wald and Lagrange multiplier tests for the regressions in equation (6) across the horizons with β_j = 1. Consistent with Berndt and Savin (1997) and Bekaert and Hodrick (2001), the Wald test substantially over-rejects the null hypothesis, while the LM test tends to under-reject the null hypothesis, especially for high significance levels. For example, for the hypothesis β_j = 1 across all four horizons, the LM test rejects only 4.6% and 0.2% of the time at the 5% and 1% levels, respectively, while the Wald test rejects the null hypothesis in 28.0% and 14.2% of the

[12] The coefficient estimates are slightly downward biased in both cases, but these results are omitted for brevity.

simulations. Moreover, while the LM test performs similarly for both the $\beta_j = 1$ and β_j equal hypotheses, the small sample properties of the Wald test are much worse for the hypothesis $\beta_j = 1$.

Appendix B: Proofs of Regression Coefficients

A. Uncovered Interest Rate Parity

The process for exchange rates is

$$\Delta s_{t,t+1} = (\pi_{t,t+1} - \pi^*_{t,t+1}) + \delta\left(E_{t+1}[r_{t+1,t+2}] - E_{t+1}[r^*_{t+1,t+2}]\right)$$
$$+ D_{t+1}\sum_{v=1}^{\tilde{W}}\left[-\delta\left(E_{t+2-v}[r_{t+2-v,t+3-v}] - E_{t+2-v}[r^*_{t+2-v,t+3-v2}]\right)\right]$$

$$D_{t+1} = \begin{cases} 1 & \text{with prob. } p \\ 0 & \text{with prob. 1-}p \end{cases}$$

where $\tilde{W}$ is the number of periods since the last crash. Rewrite the exchange rate change in terms of inflation differentials:

$$\Delta s_{t,t+1} = (\pi_{t,t+1} - \pi^*_{t,t+1}) + \delta\gamma(\pi_{t,t+1} - \pi^*_{t,t+1}) + D_{t+1}\sum_{v=1}^{\tilde{W}}\left[-\delta\gamma(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})\right]$$
$$= (1+\delta\gamma)(\pi_{t,t+1} - \pi^*_{t,t+1}) + D_{t+1}\sum_{v=1}^{\tilde{W}}\left[-\delta\gamma(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})\right]$$

and consider the UIP regression with independent variable is $i_{t,1} - i^*_{t,1} = (\theta+\gamma)(\pi_{t-1,t} - \pi^*_{t-1,t})$.

The regression coefficient is:

$$\beta_0 = \frac{\text{cov}(\Delta s_{t,t+1}, i_{t,1} - i^*_{t,1})}{\text{var}(i_{t,1} - i^*_{t,1})}$$

$$\text{var}(i_t - i^*_t) = \text{var}((\theta+\gamma)(\pi_{t-1,t} - \pi^*_{t-1,t})) = (\theta+\gamma)^2\sigma^2_{d\pi}$$

$$\text{cov}(\Delta s_{t,t+1}, i_{t,1} - i^*_{t,1}) = \text{cov}\left((1+\delta\gamma)(\pi_{t,t+1} - \pi^*_{t,t+1}) + D_{t+1}\sum_{v=1}^{\tilde{W}}\left[-\delta\gamma(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})\right]\right.$$
$$\left., (\theta+\gamma)(\pi_{t-1,t} - \pi^*_{t-1,t})\right)$$
$$= (1+\delta\gamma)\theta(\theta+\gamma)\sigma^2_{d\pi}$$
$$+ \text{cov}\left(D_{t+1}\sum_{v=1}^{\tilde{W}}\left[-\delta\gamma(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})\right], (\theta+\gamma)(\pi_{t-1,t} - \pi^*_{t-1,t})\right)$$

To compute this second term, note that (i) the inflation differentials are mean zero, so the covariance is just the expectation of the product, (ii) the dummy variable and the inflation differentials are independent of each other, and (iii) the covariance between inflation differentials at different points in time is

$$\text{cov}(\pi_{t-1,t} - \pi^*_{t-1,t}, \pi_{t-1-j,t-j} - \pi^*_{t-1-j,t-j}) = E\left[(\pi_{t-1,t} - \pi^*_{t-1,t})(\pi_{t-1-j,t-j} - \pi^*_{t-1-j,t-j})\right] = \theta^j\sigma^2_{d\pi}.$$

Therefore, the second term above is

$$\text{cov}\left(D_{t+1} \sum_{v=1}^{\tilde{W}} \left[-\delta\gamma(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v}) \right], (\theta+\gamma)(\pi_{t-1,t} - \pi^*_{t-1,t}) \right)$$

$$= -\delta\gamma(\theta+\gamma) E[D_{t+1}] E\left(\sum_{v=1}^{\tilde{W}} \left[(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})(\pi_{t-1,t} - \pi^*_{t-1,t}) \right] \right)$$

$$= -\delta\gamma(\theta+\gamma) p \sum_{n=1}^{\infty} \left(\Pr[\tilde{W} = n] \sum_{v=1}^{n} E\left[(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})(\pi_{t-1,t} - \pi^*_{t-1,t}) \right] \right)$$

$$= -\delta\gamma(\theta+\gamma) p \sum_{n=1}^{\infty} \left(p(1-p)^{n-1} \sum_{v=1}^{n} E\left[(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})(\pi_{t-1,t} - \pi^*_{t-1,t}) \right] \right)$$

$$= -\delta\gamma(\theta+\gamma) p \left[\sum_{n=1}^{\infty} \left(p(1-p)^{n-1} \left[\theta + \frac{1-\theta^{n-1}}{1-\theta} \right] \sigma^2_{d\pi} \right) \right]$$

$$= -\delta\gamma(\theta+\gamma) p \left[\theta + \frac{1}{1-\theta} - \sum_{n=1}^{\infty} \left(p(1-p)^{n-1} \frac{\theta^{n-1}}{1-\theta} \right) \right] \sigma^2_{d\pi}$$

$$= -\delta\gamma(\theta+\gamma) p \left[\theta + \frac{1}{1-\theta} - \left(\frac{p}{1-\theta} \right) \left(\frac{1}{1-(1-p)\theta} \right) \right] \sigma^2_{d\pi}$$

Putting it all back together

$$\beta_0 = \frac{\text{cov}(\Delta s_{t,t+1}, i_t - i_t^*)}{\text{var}(i_t - i_t^*)}$$

$$= \frac{(1+\delta\gamma)\theta}{\theta+\gamma} - \frac{\delta\gamma p \left[\theta + \frac{1}{1-\theta} - \left(\frac{p}{1-\theta} \right) \left(\frac{1}{1-(1-p)\theta} \right) \right]}{\theta+\gamma}$$

$$= \frac{\theta}{\theta+\gamma} + \frac{\delta\gamma\theta}{\theta+\gamma} - \frac{\delta\gamma p \left[\theta + \frac{1-p}{1-(1-p)\theta} \right]}{\theta+\gamma}$$

B. Uncovered Interest Rate Parity with Forward Interest Rate Differentials

Consider the regression on lagged forward interest rate differentials:

$$\Delta s_{t,t+1} = \beta_1 (if^{1,2}_{t-1} - if^{1,2^*}_{t-1}) + u_{t+1}$$

The dependent variable is the same as above. Under the expectations hypothesis of interest rates, the independent variable is

$$if^{1,2}_{t-1} - if^{1,2^*}_{t-1} = (\theta+\gamma)\theta(\pi_{t-2,t-1} - \pi^*_{t-2,t-1})$$

The regression coefficient is

$$\beta_1 = \frac{\text{cov}(\Delta s_{t,t+1}, if_{t-1}^{1,2} - if_{t-1}^{1,2^*})}{\text{var}(if_{t-1}^{1,2} - if_{t-1}^{1,2^*})}$$

$$\text{var}(if_{t-1}^{1,2} - if_{t-1}^{1,2^*}) = (\theta+\gamma)^2\theta^2\sigma_{d\pi}^2$$

$$\begin{aligned}\text{cov}(\Delta s_{t,t+1}, if_{t-1}^{1,2} - if_{t-1}^{1,2^*}) &= \text{cov}\Bigg((1+\delta\gamma)(\pi_{t,t+1} - \pi_{t,t+1}^*) + D_{t+1}\sum_{v=1}^{\tilde{W}}\left[-\delta\gamma(\pi_{t+1-v,t+2-v} - \pi_{t+1-v,t+2-v}^*)\right] \\ &\qquad ,(\theta+\gamma)\theta(\pi_{t-2,t-1} - \pi_{t-2,t-1}^*)\Bigg) \\ &= (1+\delta\gamma)(\theta+\gamma)\theta^3\sigma_{d\pi}^2 \\ &\quad + \text{cov}\left(D_{t+1}\sum_{v=1}^{\tilde{W}}\left[-\delta\gamma(\pi_{t+1-v,t+2-v} - \pi_{t+1-v,t+2-v}^*)\right], (\theta+\gamma)\theta(\pi_{t-2,t-1} - \pi_{t-2,t-1}^*)\right)\end{aligned}$$

The second term is

$$\begin{aligned}&\text{cov}\left(D_{t+1}\sum_{v=1}^{\tilde{W}}\left[-\delta\gamma(\pi_{t+1-v,t+2-v} - \pi_{t+1-v,t+2-v}^*)\right], (\theta+\gamma)\theta(\pi_{t-2,t-1} - \pi_{t-2,t-1}^*)\right) \\ &= -\delta\gamma(\theta+\gamma)\theta E[D_{t+1}]E\left(\sum_{v=1}^{\tilde{W}}\left[(\pi_{t+1-v,t+2-v} - \pi_{t+1-v,t+2-v}^*)(\pi_{t-2,t-1} - \pi_{t-2,t-1}^*)\right]\right) \\ &= -\delta\gamma(\theta+\gamma)\theta p\sum_{n=1}^{\infty}\left(\Pr[\tilde{W}=n]\sum_{v=1}^{n}E\left[(\pi_{t+1-v,t+2-v} - \pi_{t+1-v,t+2-v}^*)(\pi_{t-2,t-1} - \pi_{t-2,t-1}^*)\right]\right) \\ &= -\delta\gamma(\theta+\gamma)\theta p\sum_{n=1}^{\infty}\left(p(1-p)^{n-1}\sum_{v=1}^{n}E\left[(\pi_{t+1-v,t+2-v} - \pi_{t+1-v,t+2-v}^*)(\pi_{t-2,t-1} - \pi_{t-2,t-1}^*)\right]\right)\end{aligned}$$

The finite sum is different from the UIP regression above because the first inflation differential starts from *t+1* whereas the second inflation differential is at *t-1*, so there are 2 terms that lead the second differential. Splitting the sum,

$$\begin{aligned}&= -\delta\gamma(\theta+\gamma)\theta p\left[p\theta^2\sigma_{d\pi}^2 + \sum_{n=2}^{\infty}\left(p(1-p)^{n-1}\left[\theta^2+\theta+\frac{1-\theta^{n-2}}{1-\theta}\right]\sigma_{d\pi}^2\right)\right] \\ &= -\delta\gamma(\theta+\gamma)\theta p\left[p\theta^2 + (1-p)\theta^2 + (1-p)\theta + \frac{1-p}{1-\theta} - \sum_{n=2}^{\infty}\left(p(1-p)^{n-1}\frac{\theta^{n-2}}{1-\theta}\right)\right]\sigma_{d\pi}^2 \\ &= -\delta\gamma(\theta+\gamma)\theta p\left[\theta^2 + (1-p)\theta + \frac{1-p}{1-\theta} - \left(\frac{p(1-p)}{1-\theta}\right)\left(\frac{1}{1-(1-p)\theta}\right)\right]\sigma_{d\pi}^2 \\ &= -\delta\gamma(\theta+\gamma)\theta p\left[\theta^2 + (1-p)\theta + \frac{1-p}{1-\theta} - \left(\frac{p(1-p)}{1-\theta}\right)\left(\frac{1}{1-(1-p)\theta}\right)\right]\sigma_{d\pi}^2\end{aligned}$$

and putting it all back together:

$$\beta_1 = \frac{(1+\delta\gamma)\theta}{\theta+\gamma} - \frac{\delta\gamma p\left[\theta^2 + (1-p)\theta + \frac{1-p}{1-\theta} - \left(\frac{p(1-p)}{1-\theta}\right)\left(\frac{1}{1-(1-p)\theta}\right)\right]}{(\theta+\gamma)\theta}$$

$$= \frac{(1+\delta\gamma)\theta}{\theta+\gamma} - \frac{\delta\gamma p\left[\theta^2 + (1-p)\left(\theta + \frac{1}{1-\theta} - \left(\frac{p}{1-\theta}\right)\left(\frac{1}{1-(1-p)\theta}\right)\right)\right]}{(\theta+\gamma)\theta}$$

$$= \frac{\theta}{\theta+\gamma} + \frac{\delta\gamma\theta}{\theta+\gamma} - \frac{\delta\gamma p\left[\theta + \frac{1-p}{\theta}\left(\theta + \frac{1-p}{1-(1-p)\theta}\right)\right]}{\theta+\gamma}$$

Note that the term in large round brackets in the adjustment term is the same as the term in square brackets in the adjustment term in the UIP regression.

C. Exchange Rate Determination with Expected and Unexpected Interest Rate Differentials

Consider the regression:

$$\Delta s_{t,t+1} = \phi_0[(i_{t,1} - i^*_{t,1}) - (if^{1,2}_{t-1} - if^{1,2^*}_{t-1})] + \phi_1(if^{1,2}_{t-1} - if^{1,2^*}_{t-1}) + u_{t+1}$$

The dependent variable is the same as above. The second independent variable is the same as above for the regression based on forward rate differentials. The first independent variable, under the expectations hypothesis of interest rates, is

$$(i_{t,1} - i^*_{t,1}) - (if^{1,2}_{t-1} - if^{1,2^*}_{t-1}) = (\theta+\gamma)(\pi_{t-1,t} - \pi^*_{t-1,t}) - (\theta+\gamma)\theta(\pi_{t-2,t-1} - \pi^*_{t-2,t-1}) = (\theta+\gamma)(\varepsilon_{t-1,t} - \varepsilon^*_{t-1,t}).$$

The independent variables are uncorrelated, thus $\phi_1 = \beta_1$, and the other coefficient is the coefficient from a univariate regression

$$\phi_0 = \frac{\operatorname{cov}(\Delta s_{t,t+1}, (\theta+\gamma)(\varepsilon_{t-1,t} - \varepsilon^*_{t-1,t}))}{\operatorname{var}((\theta+\gamma)(\varepsilon_{t-1,t} - \varepsilon^*_{t-1,t}))}$$

$$\operatorname{var}((\theta+\gamma)(\varepsilon_{t-1,t} - \varepsilon^*_{t-1,t})) = (\theta+\gamma)^2\sigma^2_{d\varepsilon}$$

$$\operatorname{cov}(\Delta s_{t,t+1}, (\theta+\gamma)(\varepsilon_{t-1,t} - \varepsilon^*_{t-1,t})) = \operatorname{cov}\Big((1+\delta\gamma)(\pi_{t,t+1} - \pi^*_{t,t+1}) + D_{t+1}\sum_{v=1}^{\tilde{W}}\left[-\delta\gamma(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})\right], (\theta+\gamma)(\varepsilon_{t-1,t} - \varepsilon^*_{t-1,t})\Big)$$

$$= (1+\delta\gamma)\theta(\theta+\gamma)\sigma^2_{d\varepsilon} + \operatorname{cov}\Big(D_{t+1}\sum_{v=1}^{\tilde{W}}\left[-\delta\gamma(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})\right], (\theta+\gamma)(\varepsilon_{t-1,t} - \varepsilon^*_{t-1,t})\Big)$$

By construction, the shocks to the inflation process are uncorrelated with contemporaneous or lagged inflation, i.e., the only terms that matter are inflation differentials at *t+1* and *t*. The second term is therefore

$$\mathrm{cov}\left(D_{t+1} \sum_{v=1}^{\tilde{W}} \left[-\delta\gamma(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v}) \right], (\theta+\gamma)(\varepsilon_{t-1,t} - \varepsilon^*_{t-1,t}) \right)$$

$$= -\delta\gamma(\theta+\gamma)E[D_{t+1}]E\left(\sum_{v=1}^{\tilde{W}} \left[(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})(\varepsilon_{t-1,t} - \varepsilon^*_{t-1,t}) \right] \right)$$

$$= -\delta\gamma(\theta+\gamma)p\sum_{n=1}^{\infty}\left(\Pr[\tilde{W}=n] \sum_{v=1}^{n} E\left[(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})(\varepsilon_{t-1,t} - \varepsilon^*_{t-1,t}) \right] \right)$$

$$= -\delta\gamma(\theta+\gamma)p\sum_{n=1}^{\infty}\left(p(1-p)^{n-1} \sum_{v=1}^{n} E\left[(\pi_{t+1-v,t+2-v} - \pi^*_{t+1-v,t+2-v})(\varepsilon_{t-1,t} - \varepsilon^*_{t-1,t}) \right] \right)$$

$$= -\delta\gamma(\theta+\gamma)p\left[p\theta\sigma^2_{d\varepsilon} + \sum_{n=2}^{\infty}\left(p(1-p)^{n-1}[\theta+1]\sigma^2_{d\varepsilon} \right) \right]$$

$$= -\delta\gamma(\theta+\gamma)p\left[p\theta + (\theta+1)(1-p) \right]\sigma^2_{d\varepsilon}$$

$$= -\delta\gamma(\theta+\gamma)p\left[\theta+1-p \right]\sigma^2_{d\varepsilon}$$

Putting it all back together,

$$\phi_0 = \frac{\theta}{\theta+\gamma} + \frac{\delta\gamma\theta}{\theta+\gamma} - \frac{\delta\gamma p(\theta+1-p)}{\theta+\gamma}.$$

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Table 1: Preliminaries

Panel A: Summary Statistics – Exchange Rates

Exchange Rate	Mean (%)	SD (%)	1st Order Autocorr.	12th Order Autocorr.	Correlations	
USD/GBP	-1.32	11.59	0.93	-0.02	1.00	0.72
USD/DEM	0.79	12.78	0.93	0.10	0.72	1.00

Panel B: Summary Statistics – Forward Rate Differentials

$if^{j,j+1} - if^{*j,j+1}$	j	Mean (%)	SD (%)	1st Order Autocorr.	12th Order Autocorr.
US-UK	0	-1.73	1.99	0.95	0.51
	1	-1.05	1.21	0.89	0.40
	2	-1.08	1.29	0.91	0.51
	3	-0.93	1.35	0.88	0.52
	4	-0.97	1.52	0.91	0.60
US-Germ.	0	1.14	2.38	0.98	0.74
	1	1.40	1.82	0.97	0.72
	2	1.58	1.52	0.96	0.71
	3	1.57	1.46	0.96	0.71
	4	1.48	1.52	0.97	0.73

Panel C: The Forward Premium Puzzle – 1-Year Horizon

Exchange Rate	α	Std. Err.	β	Std. Err.	R^2
USD/GBP	-2.74	2.02	-0.82	0.88	2.00
USD/DEM	1.61	1.88	-0.72	0.72	1.81

Panels A and B report summary statistics (mean, standard deviation, first-order autocorrelation, twelfth-order autocorrelation, and cross correlations) for annual changes in log exchange rates and 1-year forward interest rate differentials at various horizons, sampled monthly. Panel C reports coefficient estimates, corresponding standard errors (heteroskedasticity and autocorrelation adjusted using the Newey and West (1987) method), and R^2s from the forward premium regression at the 1-year horizon

$$\Delta s_{t,t+1} = \alpha + \beta(i_{t,1} - i_{t,1}^{*}) + \varepsilon_{t,1}.$$

Exchange rate data cover 1/1980–9/2010 and interest rate data cover 1/1980–9/2009, 1/1979–9/2008, 1/1978–9/2007, 1/1977–9/2006, and 1/1976–9/2005 for horizons *j*=0,…,4, respectively, for a total of 357 monthly observations in all cases. See Section II for a detailed description of the data.

Table 2: The Expectations Hypothesis of Exchange Rates

Panel A: Regression Results

Exchange Rate	j	α	Std. Err.	β	Std. Err.	SD	P-value	R^2
USD/GBP	0	-2.74	2.02	-0.82	0.88	0.93	5.56	2.00
	1	-0.31	2.39	0.96	1.31	1.04	96.63	1.00
	2	2.34	2.16	3.37	1.01	1.18	4.95	14.10
	3	0.49	1.80	1.95	1.02	1.39	47.85	5.17
	4	1.18	1.95	2.56	0.88	1.76	36.95	11.21
USD/DEM	0	1.61	1.88	-0.72	0.72	1.00	8.44	1.81
	1	-0.12	1.96	0.65	1.21	1.11	73.48	0.85
	2	-0.28	2.17	0.68	1.30	1.26	78.61	0.65
	3	-2.38	2.37	2.01	1.39	1.48	47.84	5.32
	4	-3.91	2.37	3.17	1.33	1.87	24.22	14.19

Panel B: Hypothesis Tests

Exchange Rate	j	β	Std. Err.	Deg. of Freedom	LM Stat.	P-value	Wald Stat.	P-value
Both	0	=1		2	6.24	0.04	6.94	0.03
Both	1	=1		2	0.10	0.95	0.10	0.95
Both	2	=1		2	3.17	0.20	6.27	0.04
Both	3	=1		2	1.14	0.57	1.30	0.52
Both	4	=1		2	3.12	0.21	4.17	0.12
Both	0	-0.75	0.67	1	0.01	0.90	0.01	0.90
Both	1	0.78	1.08	1	0.06	0.81	0.06	0.81
Both	2	2.29	0.95	1	2.56	0.11	3.56	0.06
Both	3	1.97	0.85	1	0.00	0.97	0.00	0.97
Both	4	2.67	0.83	1	0.22	0.64	0.23	0.63
USD/GBP	1234	=1		4	4.58	0.33	8.48	0.08
USD/DEM	1234	=1		4	4.07	0.40	4.82	0.31
Both	1234	=1		8	7.47	0.49	17.46	0.03
USD/GBP	1234	0.24	0.70	3	4.62	0.20	6.77	0.08
USD/DEM	1234	0.96	1.30	3	3.98	0.26	4.47	0.21
Both	1234	0.64	0.55	7	7.22	0.41	17.46	0.01

Panel A reports coefficient estimates, corresponding standard errors (heteroskedasticity and autocorrelation adjusted using the Newey and West (1987) method) and R^2s from the forward premium regression (see Section II.B)

$$\Delta s_{t,t+1} = \alpha + \beta(if_{t-j}^{j,j+1} - if_{t-j}^{j,j+1^*}) + \varepsilon_{t-j,j+1},$$

using annual data sampled monthly. All regressions are run using exchange rate data over 1/1980–9/2010 (see Section II for a detailed description of the data). The columns labeled "SD" and "P-value" report simulated cross-sectional standard deviations of the estimated coefficient and two-sided P-values for the test β =1, respectively, under the Monte Carlo scheme described in Appendix A. Panel B reports tests of the hypotheses that β =1 and that the βs are equal for various combinations of exchange rates and horizons. The Lagrange Multiplier test statistics (LM Stat.) impose the relevant restrictions and the Wald test statistics (Wald Stat.) are based on the unrestricted parameter estimates. We report the restricted parameter estimate and associated standard error where relevant.

Table 3: Simulation Results from Exchange Rate Model with Constant Crash Probability

							R-squared (%)				
γ	δ	β_0	β_1	β_2	β_3	β_4	0	1	2	3	4
0.0	0	1.00	1.00	1.01	1.02	1.03	64.11	41.3	26.57	17.34	11.33
0.0	-5	1.00	1.00	1.01	1.02	1.03	64.11	41.3	26.57	17.34	11.33
0.0	-10	1.00	1.00	1.01	1.02	1.03	64.11	41.3	26.57	17.34	11.33
0.0	-15	1.00	1.00	1.01	1.02	1.03	64.11	41.3	26.57	17.34	11.33
0.0	-20	1.00	1.00	1.01	1.02	1.03	64.11	41.3	26.57	17.34	11.33
0.3	0	0.73	0.73	0.73	0.74	0.75	64.11	41.3	26.57	17.34	11.33
0.3	-5	0.06	0.21	0.39	0.61	0.80	0.03	0.22	0.50	0.77	0.86
0.3	-10	-0.60	-0.31	0.05	0.48	0.86	0.71	0.12	0.00	0.11	0.24
0.3	-15	-1.27	-0.83	-0.29	0.35	0.92	1.37	0.38	0.03	0.03	0.12
0.3	-20	-1.94	-1.35	-0.63	0.21	0.98	1.77	0.55	0.08	0.01	0.08
0.5	0	0.62	0.62	0.62	0.63	0.63	64.11	41.3	26.57	17.34	11.33
0.5	-5	-0.32	-0.12	0.14	0.44	0.71	0.41	0.03	0.03	0.20	0.33
0.5	-10	-1.26	-0.85	-0.34	0.26	0.79	1.52	0.44	0.05	0.02	0.10
0.5	-15	-2.20	-1.59	-0.82	0.07	0.87	2.04	0.68	0.12	0.00	0.05
0.5	-20	-3.14	-2.32	-1.30	-0.12	0.96	2.32	0.81	0.16	0.00	0.04
0.7	0	0.53	0.54	0.54	0.54	0.55	64.11	41.3	26.57	17.34	11.33
0.7	-5	-0.61	-0.36	-0.05	0.32	0.65	0.96	0.21	0.00	0.07	0.18
0.7	-10	-1.75	-1.25	-0.63	0.09	0.74	1.96	0.64	0.10	0.00	0.06
0.7	-15	-2.89	-2.14	-1.21	-0.13	0.84	2.36	0.83	0.17	0.00	0.03
0.7	-20	-4.03	-3.03	-1.80	-0.36	0.94	2.57	0.93	0.21	0.01	0.02

Table 3 presents regression results from the simulated exchange rate model described in equations (7)-(11). Results are based on a single simulation of 100,000 observations with an inflation persistence parameter θ = 0.8 and a constant crash probability of 7% per period. Columns 3-7 and 8-12 present coefficients and R^2s, respectively, from the forward premium regression (see Section II.B)

$$\Delta s_{t,t+1} = \alpha + \beta(if_{t-j}^{j,j+1} - if_{t-j}^{j,j+1^*}) + \varepsilon_{t-j,j+1},$$

for horizons up to 4 years.

Table 4: Simulation Results from Exchange Rate Model with Time-Varying Crash Probability

Panel A: Forward Premium Regressions

									R-squared (%)				
γ	δ	θ	$\overline{p}$	β_0	β_1	β_2	β_3	β_4	0	1	2	3	4
0.5	-10	0.80	7%	-0.41	0.27	1.03	1.82	2.61	0.21	0.06	0.53	1.06	1.41
0.4	-10	0.80	7%	-0.22	0.37	1.02	1.71	2.40	0.08	0.14	0.70	1.26	1.59
0.6	-10	0.80	7%	-0.57	0.19	1.03	1.90	2.79	0.32	0.02	0.42	0.94	1.28
0.5	**-8**	0.80	7%	-0.21	0.34	0.94	1.58	2.21	0.08	0.14	0.70	1.26	1.59
0.5	**-12**	0.80	7%	-0.62	0.21	1.10	2.05	3.00	0.32	0.02	0.42	0.94	1.28
0.5	-10	**0.75**	7%	-0.53	0.28	1.24	2.33	3.47	0.36	0.06	0.63	1.24	1.55
0.5	-10	**0.85**	7%	-0.27	0.29	0.85	1.40	1.90	0.08	0.07	0.44	0.86	1.14
0.5	-10	0.80	**6%**	-0.61	0.06	0.82	1.59	2.37	0.42	0.00	0.31	0.76	1.07
0.5	-10	0.80	**8%**	-0.27	0.46	1.22	1.96	2.74	0.10	0.18	0.81	1.34	1.67

Panel B: Bivariate Augmented Forward Premium Regressions

												R-squared (%)			
γ	δ	θ	$\overline{p}$	$\phi_{0,1}$	$\phi_{0,2}$	$\phi_{0,3}$	$\phi_{0,4}$	ϕ_1	ϕ_2	ϕ_3	ϕ_4	1	2	3	4
0.5	-10	0.80	7%	-1.63	-1.42	-1.21	-1.04	0.28	1.03	1.83	2.63	1.23	1.98	2.39	2.50
0.4	-10	0.80	7%	-1.28	-1.10	-0.92	-0.76	0.37	1.03	1.72	2.41	1.12	1.87	2.28	2.39
0.6	-10	0.80	7%	-1.94	-1.69	-1.46	-1.27	0.19	1.03	1.91	2.81	1.35	2.08	2.48	2.59
0.5	**-8**	0.80	7%	-1.18	-1.01	-0.85	-0.71	0.34	0.95	1.58	2.23	1.12	1.87	2.28	2.39
0.5	**-12**	0.80	7%	-2.09	-1.82	-1.58	-1.36	0.21	1.11	2.06	3.02	1.35	2.08	2.48	2.59
0.5	-10	**0.75**	7%	-1.58	-1.36	-1.16	-0.99	0.28	1.25	2.34	3.49	1.45	2.25	2.64	2.67
0.5	-10	**0.85**	7%	-1.74	-1.51	-1.30	-1.10	0.30	0.86	1.41	1.91	1.04	1.70	2.06	2.14
0.5	-10	0.80	**6%**	-1.80	-1.61	-1.40	-1.22	0.06	0.82	1.60	2.39	1.33	2.03	2.39	2.47
0.5	-10	0.80	**8%**	-1.57	-1.32	-1.07	-0.89	0.46	1.22	1.97	2.75	1.34	2.15	2.47	2.54

Table 4 Cont'd

Panel C: Bivariate Real Exchange Rate Regressions

γ	δ	θ	$\bar{p}$	ψ_1	ψ_2	R^2
0.5	-10	0.80	7%		-0.41	0.21
0.5	-10	0.80	7%	-0.13		7.75
0.5	-10	0.80	7%	-0.21	-2.67	13.73
0.4	-10	0.80	7%	-0.21	-2.18	13.68
0.6	-10	0.80	7%	-0.21	-3.09	13.77
0.5	**-8**	0.80	7%	-0.21	-2.01	13.68
0.5	**-12**	0.80	7%	-0.21	-3.32	13.77
0.5	-10	**0.75**	7%	-0.20	-2.56	13.72
0.5	-10	**0.85**	7%	-0.21	-2.78	13.49
0.5	-10	0.80	**6%**	-0.17	-2.68	12.34
0.5	-10	0.80	**8%**	-0.24	-2.66	15.12

Table 4 presents regression results from the simulated exchange rate model described in equations (7)-(10). Results are based on a single simulation of 100,000 observations with a time-varying crash probability given by equation (16), where the weight is set so that the average crash probability equals the value in column 4. The benchmark model is presented in the first row of each panel, and subsequent pairs of rows show deviations around this benchmark for a specific parameter, which is highlighted in bold. In Panel A, columns 5-9 and 10-14 present coefficients and R^2s, respectively, from the forward premium regression (see Section II.B)

$$\Delta s_{t,t+1} = \alpha + \beta(if_{t-j}^{j,j+1} - if_{t-j}^{j,j+1^*}) + \varepsilon_{t-j,j+1},$$

for horizons up to 4 years. In Panel B, columns 5-12 and 13-16 present coefficients and R^2s, respectively, from the bivariate augmented forward premium regression model (see Section III.D)

$$\Delta s_{t,t+1} = \alpha_j + \phi_{0,j}[(i_{t,1} - i_{t,1}^*) - (if_{t-j}^{j,j+1} - if_{t-j}^{j,j+1^*})] + \phi_j(if_{t-j}^{j,j+1} - if_{t-j}^{j,j+1^*}) + u_{t+1},$$

for horizons up to 4 years. In Panel C, columns 5-6 and 7 present coefficients and R^2s, respectively, from the bivariate real exchange rate regression model

$$\Delta s_{t,t+1} = \alpha + \psi_1 PPPD_t + \psi_2(i_{t,1} - i_{t,1}^*) + \varepsilon_{t+1}.$$

Table 5: Decomposing Interest Rate Differentials

Panel A: Regression Results

Exchange Rate	j	α	Std. err.	ϕ_j	Std. err.	$\phi_{0,j}$	Std. err.	R^2
USD/GBP	1	-1.63	2.42	0.40	1.39	-1.09	0.84	4.22
	2	0.98	2.36	2.57	1.23	-0.76	0.69	15.83
	3	-0.77	2.22	1.16	1.49	-0.66	0.88	6.43
	4	0.47	2.05	2.10	1.26	-0.34	0.81	11.53
USD/DEM	1	-0.56	1.98	0.53	1.17	-2.28	0.94	9.41
	2	-0.03	2.17	0.24	1.38	-1.01	0.67	3.74
	3	-1.71	2.41	1.32	1.58	-0.96	0.72	8.42
	4	-3.06	2.45	2.39	1.56	-0.89	0.81	16.92

Panel B: Hypothesis Tests for ϕ_j

Exchange Rate	j	ϕ_j	Std. Err.	Deg. of Freedom	LM Stat.	P-value	Wald Stat.	P-value
Both	1	=1		2	0.23	0.89	0.23	0.89
Both	2	=1		2	1.83	0.40	2.55	0.28
Both	3	=1		2	0.05	0.98	0.05	0.98
Both	4	=1		2	1.23	0.54	1.31	0.52
Both	1	0.49	1.10	1	0.01	0.92	0.01	0.92
Both	2	1.42	1.12	1	1.81	0.18	2.23	0.14
Both	3	1.24	1.14	1	0.01	0.94	0.01	0.94
Both	4	2.20	1.05	1	0.03	0.87	0.03	0.87
USD/GBP	1234	=1		4	4.76	0.31	8.37	0.08
USD/DEM	1234	=1		4	3.19	0.53	3.41	0.49
Both	1234	=1		8	6.77	0.56	16.46	0.04
USD/GBP	1234	0.99	1.17	3	4.76	0.19	7.83	0.05
USD/DEM	1234	3.02	1.17	3	3.30	0.35	3.40	0.33
Both	1234	0.54	0.72	7	6.76	0.45	16.46	0.02

Table 5 Cont'd

Panel C: Hypothesis Tests for $\phi_{0,j}$

Exchange Rate	j	$\phi_{0,j}$	Std. Err.	Deg. of Freedom	LM Stat.	P-value	Wald Stat.	P-value
Both	1	=1		2	7.94	0.02	12.41	0.00
Both	2	=1		2	8.37	0.02	9.96	0.01
Both	3	=1		2	6.81	0.03	7.50	0.02
Both	4	=1		2	6.28	0.04	5.56	0.06
Both	1	-1.52	0.81	1	2.44	0.12	2.45	0.12
Both	2	-0.90	0.60	1	0.16	0.69	0.16	0.69
Both	3	-0.88	0.70	1	0.18	0.67	0.17	0.68
Both	4	-0.63	0.72	1	0.55	0.46	0.54	0.46
USD/GBP	1234	=1		4	6.92	0.14	12.56	0.01
USD/DEM	1234	=1		4	10.58	0.03	13.95	0.01
Both	1234	=1		8	11.73	0.16	18.65	0.02
USD/GBP	1234	-0.98	0.64	3	3.51	0.32	4.54	0.21
USD/DEM	1234	-0.59	0.61	3	2.53	0.47	3.57	0.31
Both	1234	-0.95	0.57	7	7.20	0.41	10.39	0.17

Table 5 reports coefficient estimates, corresponding standard errors (heteroskedasticity and autocorrelation adjusted using the Newey and West (1987) method) and R^2s from the estimation of the augmented forward premium regression (see Section IV.B for details):

$$\Delta s_{t,t+1} = \alpha_j + \phi_{0,j}[(i_{t,1} - i_{t,1}^*) - (if_{t-j}^{j,j+1} - if_{t-j}^{j,j+1^*})] + \phi_j(if_{t-j}^{j,j+1} - if_{t-j}^{j,j+1^*}) + u_{t+1},$$

using annual data sampled monthly. All regressions are run using exchange rate data over 1/1980–9/2010 (see Section II for a detailed description of the data).

Table 6: The Information in the Contemporaneous Forward Interest Rate Differentials

Exchange Rate	j	α	Std. err.	ϕ_j	Std. err.	$\phi_{0,j}$	Std. err.	R^2
USD/GBP	1	-2.80	2.04	-0.61	1.32	-1.04	1.43	2.05
	2	-2.60	2.09	-1.09	1.13	-0.43	1.33	2.18
	3	-2.72	2.03	-0.91	1.08	-0.65	1.34	2.04
	4	-2.73	2.00	-0.95	1.05	-0.50	1.38	2.13
USD/DEM	1	1.38	1.89	-1.07	1.91	-0.59	1.17	1.87
	2	0.75	2.14	-1.32	1.26	-0.24	1.38	2.33
	3	0.82	2.17	-1.10	0.98	-0.26	1.46	2.27
	4	0.69	2.13	-1.09	0.89	-0.16	1.50	2.51

Table 6 reports coefficient estimates, corresponding standard errors (heteroskedasticity and autocorrelation adjusted using the Newey and West (1987) method) and R^2s from the estimation of the augmented forward premium regression with contemporaneous forward rate differentials (see Section IV for details):

$$\Delta s_{t,t+1} = \alpha + \phi_{0,j}[(i_{t,1} - i_{t,1}^*) - (if_t^{j,j+1} - if_t^{j,j+1^*})] + \phi_j(if_t^{j,j+1} - if_t^{j,j+1^*}) + u_{t+1},$$

using annual data sampled monthly. All regressions are run using exchange rate data over 1/1980–9/2010 (see Section II for a detailed description of the data).

Table 7: Real Exchange Rates and The Expectations Hypothesis of Exchange Rates

Panel A: Summary Statistics

Exchange Rate	Mean	SD	1st Order Autocorr.	12th Order Autocorr.	Correlations	
USD/GBP	0.46	0.14	0.98	0.65	1.00	0.71
USD/DEM	-0.46	0.16	0.98	0.70	0.71	1.00

Panel B: Regression Results

Exchange Rate	α	Std. err.	ψ_1	Std. err.	ψ_2	Std. err.	R^2
USD/GBP	-2.74	2.02			-0.82	0.88	2.00
	14.95	6.21	-0.35	0.13			18.77
	14.03	5.93	-0.38	0.12	-1.24	0.80	23.18
USD/DEM	1.61	1.88			-0.72	0.72	1.81
	-14.29	5.18	-0.33	0.12			17.49
	-16.31	4.93	-0.41	0.11	-1.70	0.54	26.37

Panel A reports summary statistics for log real exchange rates over the period 1/1980-9/2009. Panel B reports coefficient estimates, corresponding standard errors (heteroskedasticity and autocorrelation adjusted using the Newey and West (1987) method) and R^2s from the estimation of the augmented forward premium regression (see Section IV for details):

$$\Delta s_{t,t+1} = \alpha + \psi_1 q_t + \psi_2 (i_{t,1} - i^*_{t,1}) + \varepsilon_{t,1},$$

using annual data sampled monthly. All regressions are run using exchange rate data over 1/1980–9/2010 (see Section II for a detailed description of the data).

Table A.1: Monte Carlo Results

Panel A: $\beta_j = 1$

	Forward Interest Rates			Long Horizon		
j	True R^2	Mean R^2	SD R^2	True R^2	Mean R^2	SD R^2
0	4.12	6.86	7.44	4.12	6.86	7.44
1	3.24	5.92	6.80	6.79	11.28	11.73
2	2.42	5.10	6.15	8.32	13.97	14.21
3	1.64	4.32	5.43	8.96	15.31	15.46
4	0.91	3.51	4.60	8.88	15.59	15.90

Correlation of β

	Forward Interest Rates				Long Horizon			
j	1	2	3	4	1	2	3	4
0	0.86	0.68	0.52	0.36	0.94	0.86	0.77	0.68
1		0.85	0.65	0.45		0.96	0.89	0.80
2			0.83	0.57			0.97	0.89
3				0.77				0.97

Panel B: $\beta_j = 0$

	Forward Interest Rates			Long Horizon		
j	True R^2	Mean R^2	SD R^2	True R^2	Mean R^2	SD R^2
0	0.00	3.10	4.16	0.00	3.10	4.16
1	0.00	3.09	4.13	0.00	5.68	7.27
2	0.00	3.05	4.06	0.00	7.70	9.46
3	0.00	2.97	3.96	0.00	9.10	10.87
4	0.00	2.79	3.73	0.00	9.99	11.71

Correlation of β

	Forward Interest Rates				Long Horizon			
j	1	2	3	4	1	2	3	4
0	0.86	0.69	0.53	0.37	0.95	0.86	0.78	0.70
1		0.85	0.64	0.45		0.96	0.89	0.80
2			0.82	0.56			0.97	0.90
3				0.76				0.97

Panel C: Hypothesis Tests

Hypothesis		LM Test			Wald Test		
$\beta_j = 1$	Level (%)	10.00	5.00	1.00	10.00	5.00	1.00
	Rejection (%)	13.27	4.62	0.18	38.02	28.02	14.20
β_j equal	Level (%)	10.00	5.00	1.00	10.00	5.00	1.00
	Rejection (%)	12.22	4.43	0.23	26.05	16.75	6.35

Table A.1 reports the results from a Monte Carlo simulation in which we generate 100,000 replications of 428 monthly observations from a model that imposes the expectations hypothesis of interest rates and either the expectations hypothesis of exchange rates, $\beta_j = 1$, or a random walk in exchange rates, $\beta_j = 0$. These observations are then aggregated to construct samples of 357 annual, monthly overlapping observations. (See Appendix A for a detailed description and Richardson and Smith (1989) for an analysis of the benefits of using overlapping observations.) Panels A and B report statistics on the coefficient estimates and R^2s from the forward premium regressions (see Section II.B)

$$\Delta s_{t,t+1} = \alpha + \beta(if_{t-j}^{j,j+1} - if_{t-j}^{j,j+1^*}) + \varepsilon_{t-j,j+1},$$

and the long-horizon regressions, after Chinn and Meredith (2005),

$$\Delta s_{t,t+1+j} = \alpha + \beta(i_{t,j} - i_{t,j}^*) + \varepsilon_{t,j}.$$

"True" refers to the analytical (infinite sample) value, and "Mean" and "SD" refer to the mean and standard deviation of the values across the simulations. For the test statistics, Panel C reports the percent of the simulations that reject the null hypothesis at the 10%, 5%, and 1% levels under $\beta_j = 1$.