

# Investment Timing, Liquidity, and Agency Costs of Debt \*

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## Abstract

This paper examines the effect of debt and liquidity on corporate investment in a continuous-time dynamic framework. We show that due to stockholder-bondholder agency conflicts, investment thresholds are U-shaped in leverage and decreasing in liquidity. While the underinvestment problem dominates for low-liquidity firms, there is overinvestment for high-liquidity firms. In the absence of tax effects, we derive the optimal level of liquid funds that eliminates agency costs by implementing the first-best investment policy for some given capital structure. In a second step we generalize the framework by introducing a tax advantage of debt, and we show that an interior solution for liquidity and capital structure optimally trades off tax benefits and agency costs of debt.

*JEL Classification:* G13, G31, G32

*Keywords:* investment timing, real options, underinvestment, overinvestment, agency costs of debt, liquidity, capital structure

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# 1 Introduction

What are the conditions leading to under- or overinvestment of levered firms? This central question regarding the interaction of corporate investment and financing policies has been discussed in the literature for several years. The underinvestment problem is first treated explicitly by Myers (1977): When debt is in place, it can be optimal from the stockholders' perspective to forgo investment opportunities that would be favorable for an all-equity firm, since the contributed investment amount is beneficial for the bondholders as well.<sup>1</sup> A parallel stream of research concentrated on the opposite problem of overinvestment as a consequence of debt financing, usually related to the asset substitution problem based on Jensen and Meckling (1976).<sup>2</sup>

However, there are only few approaches that discuss both under- and overinvestment in a unified modelling framework. Parrino and Weisbach (1999) examine the magnitude of under- and overinvestment and their sensitivity on firm characteristics within a simulation study. These authors conclude that the agency costs of debt as such are too small to explain capital structure decisions for the average firm. However, for an entrepreneurial firm with a high-risk project, the disregard of agency costs can lead to a significant loss in firm value.

Two recent papers elaborate on the fact that agency costs of both under- and overinvestment can be mitigated if a part of the investment amount is financed by a new debt issue upon investment: Sarkar (2007) shows that depending on the relative importance of tax benefits and bankruptcy costs, the stockholders' optimal ex-post policy does generally not eliminate the agency costs of debt. Lyandres and Zhdanov (2006) separate over- from underinvestment and focus on the effect of the leverage level on the firm's investment timing. They illuminate a different rationale for overinvestment: Instead of focussing on

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<sup>1</sup>Subsequently, the underinvestment issue has been examined by many other authors, see for example Mello and Parsons (1992), who model the operating policy of a levered mine in a continuous-time dynamic approach, Mauer and Ott (2000), who analyze optimal capital structure and mitigation of agency problems, or a recent paper by Moyen (2007) for an analysis of debt maturity and underinvestment.

<sup>2</sup>One recent example is the paper by Mauer and Sarkar (2005), who analyze overinvestment in the context of a conditional debt contract.

the asset-substitution problem, they show that there can be overinvestment due to the threat of costly default, which reduces the value of waiting.<sup>3</sup>

Childs, Mauer, and Ott (2005) attribute the contrary investment situations of asset expansion and asset substitution to under- and overinvestment, respectively. However, their asset side before investment is exogenous: The firm cannot influence whether it is in a regime of asset expansion or asset substitution.

In contrast, we capture the distinction between the two situations in an analytically tractable approach by one single parameter that the firm can endogenously choose: Its level of liquid funds. We show that this is the key factor that determines the effect of existing debt on investment timing, i.e. on the decision when to exercise a real option that consists of launching an investment project. Whenever the firm's liquidity is sufficient to allow investment using internal funds only, no additional capital is needed and pure asset substitution can take place, in the sense that the risk-free liquid funds are replaced by a risky investment project. This bears incentives to overinvest as put forward by Jensen and Meckling (1976) since the stockholders are willing to invest in projects that would not be pursued from a total firm perspective. In contrast, when the firm does not have any free liquid funds, the whole investment amount has to be injected by the stockholders. This corresponds to the pure asset expansion case and induces underinvestment in the sense of Myers (1977).

In fact, already Myers (1977) suggested that a restriction on dividends, thus retaining cash for future investment, might be a way to mitigate the underinvestment problem, and he contrasted the benefits of this idea with the Jensen and Meckling (1976) problem.<sup>4</sup> However, to our knowledge this comment has not led to subsequent work dealing with under- and overinvestment that would explicitly allow the firm to vary upon debt issuance a relatively easily changeable part of its existing assets, namely its level of liquid funds, in order to endogenously mitigate agency problems.<sup>5</sup>

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<sup>3</sup> A similar argument is presented by Egami, Esteghamat, and Weng (2006).

<sup>4</sup> An empirical study by Billett, King, and Mauer (2007) shows that debt covenants, in particular dividend restrictions, can reduce agency conflicts on the exercise of growth options.

<sup>5</sup> An article by Morellec (2001) discusses the liquidity of existing assets and implications for agency

Note that the impact of liquidity on investment presented in our paper is not driven by financing constraints stemming from transaction costs or information asymmetries.<sup>6</sup> Rather we assume symmetric information, which allows issuance of fairly priced securities and excludes financing constraints. Thus, the non-existence of binding contracts on investment policy is the only friction in our model. Since we assume that the firm is managed by its sole stockholder, we can therefore focus exclusively on the implications of the level of liquid funds for the shareholder-bondholder conflict and thus for the firm's investment timing decision.<sup>7</sup>

We derive the following three implications: Firstly, the levered firm's investment threshold is U-shaped in leverage, whenever a part of the investment amount is covered by internal liquid funds. This can be explained by the fact that risk-shifting incentives are more important for small amounts of debt, but for higher leverage the underinvestment argument becomes predominant. The second implication further emphasizes the central role of liquid funds: In the absence of tax effects and for a given leverage, investment thresholds are decreasing in liquidity. There exists an optimal level of liquidity that eliminates agency costs by implementing the ex-ante optimal investment policy. Generalizing the framework by introducing tax benefits as a reason why the firm initially chooses a positive amount of debt, we derive the third implication: The importance of the level of liquidity remains valid, and there exists an interior combination of liquid funds and leverage that trades off tax benefits and agency costs of debt. It therefore should be observable that especially firms with significant growth opportunities have a target level of liquid funds, even if they are financially unconstrained.

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costs and capital structure. However the focus is not on financing investment, but on disinvestment in order to finance the firm's continued operation. An empirical study by Bates (2005) shows that the proceeds of subsidiary sales are retained more frequently if there are future growth opportunities, though this may cause agency costs of overinvestment.

<sup>6</sup> A large strand of literature, in contrast, does deal exactly with the impact of financing constraints on investment policy. An early example is the empirical study by Fazzari, Hubbard, and Petersen (1988). Recent approaches by Boyle and Guthrie (2003), Hirth and Uhrig-Homburg (2007), and Lyandres (2007) are more related to this work since they also model investment timing and therefore can capture overinvestment as well, in their case however resulting from financing constraints.

<sup>7</sup> See Opler, Pinkowitz, Stulz, and Williamson (1999) for a more detailed discussion of the possible determinants of the optimal level of liquid funds mentioned in this paragraph.

This paper is organized as follows: In Section 2 we set up our modelling framework, and we present solutions for the well-known boundary cases after investment and after default, respectively. We then solve the model in Section 3 for equity and debt values of the levered firm before investment. In Section 4 we analyze the levered firm's investment threshold, and we solve for the optimal amount of liquid funds given an exogenous capital structure and no tax effects. Then we discuss the interaction of liquid funds and capital structure in Section 5 given that there is a tax advantage of debt financing. Section 6 relates the implications of our work to existing literature. Section 7 concludes.

## 2 Model

### 2.1 Framework

We consider a partially debt-financed firm at time  $t$  that holds the perpetual rights worth  $F^o(V)$  to a project whose value  $V$  follows the geometric Brownian motion

$$dV = (\mu - \delta)Vdt + \sigma VdW \tag{1}$$

where  $\mu$ ,  $\delta$ , and  $\sigma$  are constant parameters and  $W$  is a Wiener process. The superscript  $o$  denotes the levered firm before investment, in contrast to other states of the firm that will be introduced below. Besides the project rights, the firm holds an amount  $X$  in liquid funds on a money market account. While the liquid funds remain constant until investment takes place, they generate a riskless income stream  $rX$  that is distributed to the stockholders, where  $r$  is the constant risk-free interest rate.<sup>8</sup> This may be interpreted as a debt covenant preventing dividend payments to drive the firm's cash holdings below  $X$ .

Note that the firm does not have any further assets in place besides the project rights and the liquid funds. While this may seem to be a rather reduced model, one could think

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<sup>8</sup> At this point, we do not consider taxes on interest income from the liquid funds.

of some existing assets that are not explicitly modelled since their dynamics are only of second-order importance for the investment timing problem analyzed here.<sup>9</sup> The model also describes a realistic situation for an entrepreneurial firm in its start-up phase: After the firm has been founded by entrepreneurs who have a unique idea, they are then deciding upon the best financing policy for their project rights. They will issue a certain amount of debt and become the stockholders of a levered firm. Moreover, they may provide liquid funds to the firm in order to cover a part of the future investment expenses.

At any point in time, the stockholders have the option to exercise the project rights. The investment requires a total amount of  $I$ , and we assume  $X \in [0, I]$ . Therefore the stockholders have to provide the missing amount  $I - X$  needed to launch the project and to change the firm's asset side from  $F^o(V) + X$  to  $V$ . After investment the project provides a cash-flow stream of  $\delta V$ , therefore there are opportunity costs of waiting, and the firm will invest as soon as a critical project value  $V_I^o$ , the investment threshold, is exceeded. This threshold determines the firm's investment timing policy, since investment will *ceteris paribus* take place earlier given that the firm decides to invest at a lower threshold.

The extreme case of  $X = 0$  can be described as pure *asset expansion*: The whole investment amount has to be contributed by the stockholders, which leads to a substantial increase in total firm value. In contrast, the case of  $X = I$  can be described as pure *asset substitution*: There is no capital inflow upon investment, but the riskless returns of the liquid funds are substituted by the risky project returns. Besides these two extreme cases, we can model a wide variety of investment conditions for intermediate values of  $X \in (0, I)$ .

The debt structure consists of one perpetual coupon bond with a continuous coupon stream of  $C$ . While we first take the debt structure as given, we will later also discuss the debt issuance decision. Assuming a simple tax environment where coupon payments are tax-deductible at the firm level, the after-tax debt service is  $(1 - \tau)C$ , where  $\tau$  is the

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<sup>9</sup> A related article by Mauer and Sarkar (2005) models a similarly reduced firm consisting only of an investment option and a conditional debt contract.

corporate tax rate.<sup>10</sup> Debt service is partly covered by the stockholders' income stream  $rX$  from the liquid funds. However, the remaining amount of

$$(1 - \tau)C - rX, \quad (2)$$

which is constant over time as long as the firm is in state  $o$ , has to be contributed by the stockholders. The total value of equity  $E^o(V)$  and debt  $D^o(V)$  is

$$E^o(V) + D^o(V) = F^o(V) + X + T^o(V), \quad (3)$$

where  $T^o(V)$  denotes the value of tax benefits. Note that there will never be a reason for stockholders to default before investment if the income from the firm's liquid funds is sufficient to cover the required payment, i.e. if (2) is negative.

However, if (2) is positive, then the firm declares bankruptcy as soon as the stockholders decide to no longer provide the necessary payments. This is the case as soon as a critical project value of  $V_B^o$  is underrun. Then the former stockholders have no more rights or obligations in the firm, and the former bondholders receive all the firm's assets worth

$$E^e(V) = F^e(V) + X. \quad (4)$$

The superscript  $e$  is used to indicate the state of the all-*equity* firm before investment. Note that we model no direct costs of bankruptcy, which is only for the sake of simplicity and could be easily incorporated in our framework. Still, the value of the project rights  $F^e(V)$  generally differs from  $F^o(V)$  because the former bondholders, now stockholders of an all-equity firm, will adopt an investment timing policy  $V_I^e$  that can be different from the levered firm's policy  $V_I^o$ .

In contrast, if the critical project value  $V_I^o$  is exceeded before bankruptcy has been declared, the project is launched. After investment the firm's assets consist solely of the

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<sup>10</sup> Similar to Leland (1994) and Mauer and Sarkar (2005), we assume that the firm always receives the tax benefits when it is solvent, regardless of its current operating income.

project  $V$  providing a cash-flow stream of  $\delta V$ , which the stockholders receive as a dividend after the net coupon stream, i.e.

$$\delta V - (1 - \tau)C. \quad (5)$$

In case of  $\delta V - (1 - \tau)C < 0$ , the stockholders have to contribute the remaining part of the coupon in order to avoid triggering default. The firm's balance in state  $i$  is

$$E^i(V) + D^i(V) = V + T^i(V), \quad (6)$$

where the superscript  $i$  denotes the state of the firm after *investment*. Once the investment project has been launched, the evolution of the asset value follows an exogenous process given by (1), and the only decision remaining to the stockholders is to declare bankruptcy by not paying the coupons anymore. This will be the case as soon as a critical project value of  $V_B^i$  is underrun. Similar to the new investment policy in state  $e$ , the default policy  $V_B^i$  after investment can be different from the default policy before investment  $V_B^o$ .

When the firm after investment (state  $i$ ) declares bankruptcy, the former bondholders take over the assets worth  $V$  and become new stockholders of an all-equity firm, therefore we have

$$E^{ie}(V) = V,$$

where the superscript  $ie$  denotes the state of the all-*equity* firm after *investment*. We end up in the same situation when the all-equity firm (state  $e$ ) eventually launches the investment project. However, note that in the latter case the former bondholders have to provide the missing amount  $I - X$  for the transition from state  $e$  to  $ie$ , while this has been the original stockholders' responsibility at the transition from state  $o$  to  $i$ .

The transitions from one state to another contingent on the evolution of the project value  $V$  are visualized in Figure 1.

[Figure 1]

## 2.2 Basic Claims

In order to derive solutions for debt and equity in the different states, three basic cash-flow types have to be evaluated, which will form the building blocks of all the solutions in the following subsections.<sup>11</sup> Once their values are given, debt and equity can be viewed as a portfolio of these three basic claims, which allows convenient representations and insightful interpretations.

To define the cash flows of our basic claims, we assume that the current project value  $V$  is between a lower bound of  $V_B$ , interpreted as a bankruptcy threshold, and an upper bound of  $V_I$ , interpreted as an investment threshold, with  $V_B < V_I$ . A lower bound of  $V_B = 0$  means that default can never occur, which is due to the properties of the stochastic process (1). Similarly, an upper bound of  $V_I = \infty$  means that there is no investment opportunity.

We denote by  $\mathbf{B}_I(V; V_B, V_I)$  the value of a claim that pays off 1 as soon as  $V_I$  is reached and becomes worthless at  $V_B$ . Besides that it yields no intermediate payments. Given that the risk inherent in  $V$  can be replicated by traded securities, it can be shown that  $\mathbf{B}_I$  satisfies the ordinary differential equation

$$\frac{1}{2}\sigma^2V^2(\mathbf{B}_I)_{VV} + (r - \delta)V(\mathbf{B}_I)_V = r\mathbf{B}_I \quad \forall V \in [V_B, V_I]$$

with  $(\mathbf{B}_I)_V$  and  $(\mathbf{B}_I)_{VV}$  denoting first and second derivatives with respect to  $V$ , respectively. Solving subject to the boundary conditions  $\mathbf{B}_I(V_B; V_B, V_I) = 0$  and  $\mathbf{B}_I(V_I; V_B, V_I) = 1$  yields the solution

$$\mathbf{B}_I(V; V_B, V_I) = \begin{cases} 0, & \text{if } V_I = \infty \\ \left(\frac{V}{V_I}\right)^{\lambda_1}, & \text{if } V_B = 0 \text{ and } V < V_I < \infty \\ \frac{(V_B)^{\lambda_2} \cdot V^{\lambda_1} - (V_B)^{\lambda_1} \cdot V^{\lambda_2}}{(V_I)^{\lambda_1} (V_B)^{\lambda_2} - (V_I)^{\lambda_2} (V_B)^{\lambda_1}}, & \text{if } V_B > 0 \text{ and } V_B < V < V_I < \infty \end{cases}$$

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<sup>11</sup> This can be seen as an time-homogenous extension of Ericsson and Reneby (1998), who present building blocks for bond valuation given only a lower (bankruptcy) threshold, to a situation with both a lower and an upper bound, where the latter will be the investment threshold in our context.

with

$$\lambda_1 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{1}{2} - \frac{r - \delta}{\sigma^2}\right)^2} \quad (7)$$

and

$$\lambda_2 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} - \sqrt{\frac{2r}{\sigma^2} + \left(\frac{1}{2} - \frac{r - \delta}{\sigma^2}\right)^2}. \quad (8)$$

Similarly,  $\mathbf{B}_B(V; V_B, V_I)$  denotes the value of a claim that pays off 1 upon reaching  $V_B$ , however becomes worthless when the project value hits  $V_I$ . Besides that it also yields no intermediate payments. It satisfies the same differential equation as  $\mathbf{B}_I$ , while the boundary conditions are  $\mathbf{B}_B(V_B; V_B, V_I) = 1$  and  $\mathbf{B}_B(V_I; V_B, V_I) = 0$ . Its value is

$$\mathbf{B}_B(V; V_B, V_I) = \begin{cases} 0, & \text{if } V_B = 0 \\ \left(\frac{V}{V_B}\right)^{\lambda_2}, & \text{if } V_I = \infty \text{ and } 0 < V_B < V \\ \frac{(V_I)^{\lambda_1} \cdot V^{\lambda_2} - (V_I)^{\lambda_2} \cdot V^{\lambda_1}}{(V_I)^{\lambda_1} (V_B)^{\lambda_2} - (V_I)^{\lambda_2} (V_B)^{\lambda_1}}, & \text{if } V_I < \infty \text{ and } 0 < V_B < V < V_I. \end{cases}$$

The third claim with value  $\mathbf{B}_O(V; V_B, V_I)$  yields a stream of the risk-free rate  $r$  as long as the project value stays in between  $V_B$  and  $V_I$ . It can be shown to be worth

$$\mathbf{B}_O(V; V_B, V_I) = 1 - \mathbf{B}_I(V; V_B, V_I) - \mathbf{B}_B(V; V_B, V_I). \quad (9)$$

Note that in order to implement a strategy that yields the risk-free rate as long as the project value remains within the interval  $(V_B, V_I)$  and a payoff of 1 upon reaching either bound, a capital investment of one monetary unit is needed. Therefore the three basic claims add up to 1.

These three basic claims can now be used in order to solve the model backwards. While the equity value in the *ie* state is already known to be  $V$ , we will first give the solutions to the *e* and *i* states, which are well-known from the literature. Afterwards we will use them to derive the values of equity and debt of the levered firm before investment (state *o*), which are new to this paper.

### 2.3 All-Equity Firm's Investment Decision

The assets of the firm after default (state  $e$ ) as given by (4) consist of the project rights  $F^e(V)$  and the liquid funds  $X$ . Since the firm after default corresponds to an all-equity firm with financially unconstrained stockholders, the firm's liquid funds do not have an impact on the investment decision, and the project rights can be valued in a classical real-option framework.<sup>12</sup> Before investment, they can be interpreted as a portfolio consisting of  $(V_I^e - I)$  units of the basic claim  $\mathbf{B}_I$  yielding one monetary unit upon investment. The resulting investment option value is

$$F^e(V) = \begin{cases} (V_I^e - I) \cdot \mathbf{B}_I(V; 0, V_I^e), & \text{if } 0 \leq V < V_I^e \\ V - I, & \text{if } V \geq V_I^e. \end{cases} \quad (10)$$

The first-order condition

$$F_V^e(V_I^e) = 1 \quad (11)$$

ensures that immediate investment is indeed optimal above the critical project value  $V_I^e$ . Solving for this investment threshold, we get

$$V_I^e = \frac{\lambda_1}{\lambda_1 - 1} \cdot I. \quad (12)$$

### 2.4 Levered Firm after Investment

Similarly, there are solutions in the literature for the values of equity and debt of the levered firm after investment (state  $i$ ), when the firm's liquid funds have been used up in order to launch the investment project, and there is no more optionality on the asset side.<sup>13</sup>

We start with the equity value  $E^i(V)$ . The stockholders' position after investment consists of holding the project value and paying the net coupon stream until default, when they

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<sup>12</sup>See McDonald and Siegel (1986) and Dixit and Pindyck (1994).

<sup>13</sup>See Black and Cox (1976) and Leland (1994).

also lose their claim to the project value. Therefore the solution to  $E^i(V)$  is

$$E^i(V) = \begin{cases} 0, & \text{if } 0 \leq V \leq V_B^i \\ V - V_B^i \cdot \mathbf{B}_B(V; V_B^i, \infty) - \frac{(1-\tau)C}{r} \cdot \mathbf{B}_O(V; V_B^i, \infty), & \text{if } V > V_B^i \end{cases} \quad (13)$$

The first-order condition

$$E_V^i(V_B^i) = 0 \quad (14)$$

ensures that the bankruptcy threshold  $V_B^i$  indeed maximizes equity value. It leads to the bankruptcy threshold

$$V_B^i = \frac{\lambda_2}{\lambda_2 - 1} \cdot \frac{(1 - \tau)C}{r}. \quad (15)$$

For a project value  $V$  being above the bankruptcy threshold  $V_B^i$ , the debt value  $D^i(V)$  can similarly be interpreted as a portfolio of a claim that provides the payment  $V_B^i$  at default, and a claim that pays a stream of  $C$  until default. Therefore the solution to  $D^i(V)$  is

$$D^i(V) = \begin{cases} V, & \text{if } 0 \leq V \leq V_B^i \\ V_B^i \cdot \mathbf{B}_B(V; V_B^i, \infty) + \frac{C}{r} \cdot \mathbf{B}_O(V; V_B^i, \infty), & \text{if } V > V_B^i. \end{cases} \quad (16)$$

Note that there is no new first-order condition – the bankruptcy threshold  $V_B^i$  is chosen by the stockholders and thus given by (15).

### 3 Levered Firm before Investment

Having presented solutions for the  $e$  and  $i$  states, and thus for the boundaries of the  $o$  state, we are now ready to derive the values of equity and debt of the levered firm before investment.

For the equity value  $E^o(V)$  of the levered firm before investment, the first boundary

condition, namely the one at  $V_I^o$ , is

$$E^o(V_I^o) = E^i(V_I^o) - (I - X). \quad (17)$$

This means that upon investment the stockholders have to provide an amount  $I - X$  and afterwards hold a position worth  $E^i(V_I^o)$  according to (13).

When considering the default decision, we have to distinguish two cases: If the stockholders' net payment given by (2) is negative, there is no default before investment, i.e.  $V_B^o = 0$ . Then debt becomes virtually riskless for very low project values, and the second boundary condition is

$$\lim_{V \rightarrow 0} E^o(V) = X - \frac{(1 - \tau)C}{r} > 0.$$

However in the second case, if the stockholders' net payment given by (2) is positive, there will be an endogenous default decision characterized by a different second boundary condition

$$E^o(V_B^o) = 0. \quad (18)$$

This means that the stockholders receive no payment at default.

In both cases, the equity value can be interpreted as a portfolio consisting of a cash-flow stream of  $rX - (1 - \tau)C$  while neither investment nor default occurs, and a payment of  $E^i(V_I^o) - (I - X)$  upon investment. Using the basic claims  $\mathbf{B}_O$  and  $\mathbf{B}_I$ , we derive the following closed-form expression for  $E^o(V)$ :

$$E^o(V) = \begin{cases} 0, & \text{if } 0 < V \leq V_B^o \\ \left(X - \frac{(1-\tau)C}{r}\right) \cdot \mathbf{B}_O(V; V_B^o, V_I^o) + \\ (E^i(V_I^o) - (I - X)) \cdot \mathbf{B}_I(V; V_B^o, V_I^o), & \text{if } V_B^o < V < V_I^o \\ E^i(V) - (I - X), & \text{if } V \geq V_I^o \end{cases} \quad (19)$$

In order to derive the optimal investment and bankruptcy thresholds, we have to consider

the first-order conditions at the boundaries. The condition at  $V_I^o$  is

$$E_V^o(V_I^o) = E_V^i(V_I^o), \quad (20)$$

which ensures that the resulting investment threshold  $V_I^o$  indeed maximizes the value of equity. Note that we thus assume that the firm follows the *second*-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. In Section 5 we will compare this to the *first*-best investment policy, i.e. the policy that maximizes the sum of equity and debt values. If default is possible at all (i.e. if (2) and thus  $V_B^o$  is positive), the second first-order condition

$$E_V^o(V_B^o) = 0 \quad \forall V_B^o > 0 \quad (21)$$

ensures that the resulting bankruptcy threshold  $V_B^o$  also maximizes the value of equity. In the case of  $V_B^o = 0$ , we do not need a second first-order condition.

On the other hand, the debt value  $D^o(V)$  has to satisfy a first boundary condition at  $V_I^o$  being

$$D^o(V_I^o) = D^i(V_I^o).$$

We distinguish again the two cases for  $V_B^o$ : If the stockholders' net payment given by (2) is negative, we have  $V_B^o = 0$  and the second boundary condition is

$$\lim_{V \rightarrow 0} D^o(V) = \frac{C}{r}.$$

In contrast, if (2) is positive, there is a different second boundary condition, namely

$$D^o(V_B^o) = E^e(V_B^o).$$

So the debt value for  $V_B^o < V < V_I^o$  corresponds to a portfolio that consists of the right to receive  $E^e(V_B^o)$  upon default, a stream of  $C$  while neither default nor investment occurs,

and  $D^i(V_I^o)$  upon investment. The solution for  $D^o(V)$  is

$$D^o(V) = \begin{cases} E^e(V), & \text{if } 0 \leq V \leq V_B^o \\ E^e(V_B^o) \cdot \mathbf{B}_B(V; V_B^o, V_I^o) + \frac{C}{r} \cdot \mathbf{B}_O(V; V_B^o, V_I^o) + \\ D^i(V_I^o) \cdot \mathbf{B}_I(V; V_B^o, V_I^o), & \text{if } V_B^o < V < V_I^o \\ D^i(V), & \text{if } V \geq V_I^o \end{cases} \quad (22)$$

For the no-default case, the first case disappears, and the second case reduces to

$$\frac{C}{r} \cdot \mathbf{B}_O(V; V_B^o, V_I^o) + D^i(V_I^o) \cdot \mathbf{B}_I(V; V_B^o, V_I^o),$$

since the right to receive  $E^e(V_B^o)$  upon default has no value.

Note that there are no new first-order conditions, since it is the stockholders' exclusive right to choose the investment and bankruptcy policies that are value-maximizing from their perspective. Therefore, both  $V_B^o$  and  $V_I^o$  result from the maximization of equity value (19), captured by the conditions (20) and (21).

## 4 Underinvestment, Overinvestment, and Liquidity

In this section, we first discuss how the effect of leverage on the firm's investment policy depends on the liquid funds. In a second step, we take the firm's leverage as given and analyze the implications of the firm's liquid funds as such on the investment policy. For both cases, we also discuss the sensitivity of our results on project risk. Throughout the current section, we abstract from tax effects ( $\tau = 0$ ), which implies  $T^i(V) = T^o(V) = 0$ . This allows us to focus solely on agency costs, i.e. the value effect of suboptimal investment. In Section 5 we will then discuss the interaction of liquidity and capital structure choice given that there is a positive tax rate ( $\tau > 0$ ), which induces an advantage of debt financing and therefore a reason to initially issue debt.

When solving for the equity value  $E^o(V)$  in (19), we have to determine at the same time

the investment and bankruptcy policies of the levered firm before investment,  $V_I^o$  and  $V_B^o$ . In doing so we assume throughout this section that the levered firm follows the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. Since we do not consider tax effects in this section, the first-best policy would be to choose the investment threshold  $V_I^e$  of the all-equity firm. Therefore we compare the levered firm's investment policy to that of an all-equity firm. Note that the investment threshold  $V_I^e$  of the all-equity firm, given by the closed-form solution (12), is not dependent on the firm's liquid funds  $X$ . In contrast, the investment policy of the levered firm crucially depends on  $X$ .

In order to link the investment threshold and thus the investment timing decision of a levered firm to the notions of under- and overinvestment, we introduce the following definitions:<sup>14</sup> Whenever the investment threshold  $V_I^o$  of the levered firm is higher than the benchmark investment threshold  $V_I^e$  of an all-equity firm, investment will *ceteris paribus* take place later, which we regard as underinvestment. On the other hand, if the levered firm chooses an investment threshold that is lower than the benchmark, we speak of overinvestment.

## 4.1 Effect of Leverage on Investment Policy

We first discuss the effect of leverage, measured by the debt coupon  $C$ , on the firm's investment policy. We take as a starting point that an unlevered firm ( $C = 0$ ) follows the investment policy  $V_I^e$  regardless of its level of liquid funds. However, Figure 2 illustrates that the effect of positive leverage does depend on the firm's cash holdings. Our numerical example uses a set of parameters given in Table 1. The absolute level of the project investment cost  $I$  is just a scaling factor. The rather high volatility parameter  $\sigma$  and the high cash-flow rate  $\delta$  relative to the riskless interest rate  $r$  can be motivated by the fact that we are interested particularly in high-risk projects as they are typical for entrepreneurial firms. In particular the effect of varying project risk  $\sigma$  will further be analyzed below.

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<sup>14</sup> These definitions can similarly be found e.g. in Childs, Mauer, and Ott (2005) and Lyandres and Zhdanov (2006).

Table 1: Parameter values used in numerical examples.

Parameter	Value
Project investment cost	$I = 100$
Project value volatility	$\sigma = 50\%$
Project cash-flow rate	$\delta = 5\%$
Riskless interest rate	$r = 5\%$

[Figure 2]

For  $X = 0$  we observe the pure asset expansion case: The stockholders are most concerned about the fact that the amount that they have to contribute to the firm also benefits the bondholders. Therefore they are reluctant to launch the investment project unless the project value is sufficiently favorable. As a consequence, the firm requires a higher project value for investment than an all-equity firm ( $\frac{V_I^o}{V_I^e} > 1$ ). Thus the firm follows a policy that can be described as underinvestment, similar to the problem discussed by Myers (1977). This effect becomes more and more pronounced with increasing leverage. Therefore the investment threshold rises more and more above the unlevered firm's level of  $V_I^e$ .

Next we discuss the case  $X = I$ , which is the pure asset substitution case first treated by Jensen and Meckling (1976). The value of equity makes up only a part of the total firm value when there is debt in the firm. Therefore the stockholders relatively bear less risk than the owners of an all-equity firm when they substitute the riskless returns of the liquid funds by the risky project returns. The stockholders should then be willing to engage in risk-shifting and invest even for a lower project value than an all-equity firm ( $\frac{V_I^o}{V_I^e} < 1$ ). Thus they follow a policy that can be described as overinvestment. The more debt there is in the firm, the more favorable this behavior is to the stockholders. Therefore worse and worse projects are accepted for increasing leverage. The investment threshold is strictly decreasing in  $C$  until it hits the bankruptcy threshold. In this degenerate case (and for any even higher coupons  $C$ ) the  $o$  state disappears completely.

To summarize, for either of these pure cases of under- and overinvestment, respectively, we observe that the distortions in investment policy resulting from stockholder-bondholder

conflicts are increasing in the firm's leverage measured by the debt coupon  $C$ . This result is consistent with the simulation study by Parrino and Weisbach (1999). However, these authors explain under- and overinvestment in a static framework by different project characteristics, while their project is financed entirely with equity.

Both cases contain another effect that is isolated by Lyandres and Zhdanov (2006): The loss of the option value (from the stockholders' perspective) in the event of default induces early exercise in order to increase equity value, i.e. the  $X = 0$  threshold is rising less steeply and the  $X = I$  threshold is falling faster due to this additional effect.

The combination of the  $X = 0$  and the  $X = I$  cases leads to an investment threshold that is U-shaped in leverage for any interior level of liquid funds. For a small debt coupon, the advantage of shifting risk to the bondholders dominates, and therefore investment takes place at project values below  $V_I^e$ . However, for increasing coupon levels default becomes more likely, and so does the stockholders' risk to provide value to the firm that is beneficial mainly to the bondholders later on. As this motivation for underinvestment becomes more and more important, the investment threshold for intermediate  $X$  eventually reaches its minimum and starts rising again. For sufficient leverage even the all-equity threshold  $V_I^e$  is exceeded.

From a practical perspective, the intermediate case where one share of the amount required for investment is covered by the liquid funds that are already in the firm, and the remaining share has to be injected upon investment, might be the most realistic one. Therefore we conclude that also the U-shape of the investment threshold in leverage is the most realistic description of investment behavior. To our knowledge, this is an essentially new contribution of our paper. It results from the fact that we account for the role of the firm's liquid funds for both over- and underinvestment.

## 4.2 Effect of Liquid Funds on Investment Policy

Now we analyze the implications of the firm's liquid funds as such on the investment policy. We claim that the investment threshold  $V_I^o$  of the levered firm should be decreasing

in the firm's liquid funds  $X$ : For low-liquidity firms, the properties of asset expansion and underinvestment ( $\frac{V_I^o}{V_I^e} > 1$ ), explained above in its pure form for  $X = 0$ , are predominant. On the other hand, for high-liquidity firms the properties of asset substitution and overinvestment ( $\frac{V_I^o}{V_I^e} < 1$ ) found in its pure form for  $X = I$  are predominant. Consequently, we expect that there will be an intermediate level  $X_0^*$  of liquid funds (the subscript 0 indicates the  $\tau = 0$  case discussed throughout Section 4) for which the levered firm follows the all-equity firm's investment policy ( $\frac{V_I^o}{V_I^e} = 1$ ).

[Figure 3]

Figure 3 shows that the levered firm's investment threshold indeed has the proposed shape, i.e. it is decreasing in the firm's liquid funds  $X$ . Again we use the set of parameters given in Table 1, and the debt coupon rate is set to  $C_0 = 27$ . The effect shown in Figure 3 for the whole range of possible  $X$  values is also visible in Figure 2 for this fixed  $C_0$  and selected values of  $X$ . Moreover, Figure 2 reveals that for  $X = I$ , the coupon level  $C_0$  is indeed in the range where the investment threshold coincides with the bankruptcy threshold, which is where the threshold is pulled down to at the right margin of Figure 3. There is an optimal amount of liquid funds  $X_0^* = 64.51$  for which the levered firm follows the all-equity firm's investment policy ( $V_I^o = V_I^e$ ). Otherwise, the firm chooses an investment policy that is suboptimal from a first-best perspective.

### 4.3 Effect of Project Risk on Investment Policy

Intuitively, we should expect that the stockholders' incentives to overinvest are increasing in project risk, since risk-shifting then becomes even more attractive. However, would this also mean that for cases in which the investment threshold is above the first-best level, the underinvestment problem is mitigated by increasing project risk?

[Figure 4]

Indeed, Figure 4 shows that the levered firm's investment threshold relative to the all-equity firm's threshold is decreasing in the project value volatility  $\sigma$  both in areas where there is overinvestment and underinvestment, respectively. This relation shown for the

case of  $X = X_0^*$  and varying  $C$  in the first graph of Figure 4 holds similarly for any other level of liquid funds, including the two extreme cases of  $X = 0$  and  $X = I$ : Even for  $X = 0$  where there is underinvestment for any positive amount of debt (see Figure 2), the underinvestment problem is mitigated for increasing project risk.

Besides further confirming the negative relation of project risk and investment threshold, the second graph of Figure 4 for the case of  $C = C_0$  and varying  $X$  allows the following interpretation: Given a certain leverage level measured by  $C$ , the optimal amount of liquid funds  $X^*$  needed in order to implement the first-best investment policy  $V_I^e$  (i.e. the amount of liquid funds that yields  $V_I^o/V_I^e = 1$ ) is decreasing in project risk: While the stockholders' overinvestment incentives are increasing in project risk, their desire can be tamed by increasing the amount  $I - X$  that they have to contribute to the investment. Therefore reducing  $X$  makes the stockholders more reluctant to invest early in risky projects. This can be used to counterbalance the effect of increasing project risk.

Note at this point that the preceding discussion has always focussed on the relative investment threshold  $V_I^o/V_I^e$ . However, the investment threshold  $V_I^e$  of an all-equity firm is itself strictly increasing in project risk  $\sigma$ . That is the usual behavior expected from option pricing theory, since the value of waiting is increasing in  $\sigma$ , but it is the contrary of the sensitivity of the relative investment threshold  $V_I^o/V_I^e$ . Examining directly the absolute investment threshold  $V_I^o$  shows a more diverse picture: For low liquid funds, it is also increasing in  $\sigma$ . If e.g.  $X = 0$ , the entire investment amount has to be provided by the stockholders. Therefore the incentives are exactly those known from option pricing theory. The relative investment threshold  $V_I^o/V_I^e$  is still decreasing in  $\sigma$  due to the fact that the increase of  $V_I^e$  in  $\sigma$  is more pronounced than that of  $V_I^o$ . For the other extreme case  $X = I$ , the stockholders have risk-shifting incentives, since investment does not only mean to give up the value of waiting, but also to transform riskless into risky assets. These incentives are obviously increasing in project risk, and they can be more important than the increasing value of waiting known from option pricing theory. Therefore, even the absolute investment threshold  $V_I^o$  of the levered firm can be decreasing in  $\sigma$  in this case.

## 4.4 Agency Costs

Now we analyze the agency costs that the firm suffers due to a suboptimal investment policy induced by existing debt. In this context the term 'suboptimal' refers to the total firm's perspective. We determined the firm's policy in Section 3 to be optimal from the stockholders' point of view ex-post, i.e. after debt is in place. However, ex-ante, i.e. before debt is in place, it would be in the interest of the stockholders if they could credibly commit themselves to the investment policy of an all-equity firm, since at that point in time, the debt contract will be set up according to the assumed investment policy. Therefore ex-ante the stockholders maximize the sum of equity and debt value, i.e. total firm value, and it would be suboptimal not to commit to the all-equity firm's investment policy. Consequently, agency costs arise since we assume that this ex-ante commitment is not possible.

We define agency costs  $AC$  for a tax rate of  $\tau = 0$  as the difference in total firm value between an all-equity firm and a levered firm:

$$AC_{\tau=0} = E^e(V) - [E^o(V) + D^o(V)] = F^e(V) - F^o(V). \quad (23)$$

They can then be understood as the part of all-equity firm value that is lost due to the suboptimal investment policy. Using (3), (4), and  $T^o(V) = 0$  for  $\tau = 0$ , we get the last expression, which illustrates that the agency costs for  $\tau = 0$  exactly consist of the loss in option value due to the deviation from the all-equity investment policy.

As elaborated in Section 4.2, the nature of investment distortions, and thus also agency costs crucially depend on the firm's liquid funds: A low-liquidity levered firm chooses a higher investment threshold than an all-equity firm, resulting in agency costs of underinvestment. On the other hand, the investment threshold of a high-liquidity levered firm is lower compared to that of an all-equity firm, resulting in agency costs of overinvestment. Only for an intermediate level of liquid funds  $X_0^*$  there are no agency costs.

[Figure 5]

Figure 5 shows the agency costs of debt for an initial project value of  $V_0 = 0.75V_I^e = 319.92$ , which is also indicated by a horizontal line in Figure 3. Comparing to that figure, it can be seen that indeed only for  $X = X_0^* = 64.51$  there are no agency costs. For lower levels of liquid funds, i.e.  $X < X_0^*$ , the investment threshold  $V_I^o$ , and thus the agency costs of underinvestment, are both increasing with distance from  $X_0^*$ .

On the other hand, for  $X > X_0^*$  the investment threshold  $V_I^o$  is decreasing with distance from  $X_0^*$ , and consequently the agency costs of overinvestment are increasing. However, since we assume an initial project value of  $V_0 = 0.75V_I^e < V_I^e$ , a new situation arises if the firm's liquid funds are high enough that the investment threshold  $V_I^o$  falls below the initial project value  $V_0$ . Going back to Figure 3, we observe that this happens for  $X > 83.83$ . Then immediate investment will take place right after debt issuance, while the all-equity firm would prefer to postpone the investment. In all of these states, we do no longer have possible future overinvestment, but actual overinvestment, and thus for all  $X > 83.83$  the agency costs of overinvestment remain at

$$F^e(V) - (V - I),$$

which is a constant value and not dependent on  $X$ .

A similar result arises for an initial project value  $V_0 > V_I^e$ , for which the all-equity firm invests immediately: We will examine that below a certain level of liquid funds  $X$  the investment threshold  $V_I^o$  will exceed  $V_0$  and underinvestment takes place. Then the agency costs of underinvestment are

$$V - I - F^o(V).$$

They are increasing for decreasing liquid funds  $X$ , since  $F^o(V)$  becomes less valuable with the distance of  $X$  and  $X_0^*$ . In contrast, for sufficiently high  $X$  we have  $V_0 \geq V_I^o$ , and both the levered firm and the all-equity firm invest immediately. In that case there are no agency costs.

## 5 Optimal Liquidity and Capital Structure Choice

Up to now, we have taken the firm's capital structure as given. Although we have analyzed the effect of leverage on investment in Section 4.1, we have not discussed how much debt should optimally be issued. In fact, our analysis throughout Section 4 omits tax effects, with the consequence that there is no model-endogenous reason at all to initially issue debt. In this context, the easiest way to avoid agency costs of debt is to remain an all-equity firm.

Therefore we now generalize our analysis: We show that when there is a positive tax rate  $\tau > 0$ , and thus a reason to issue debt due to tax benefits, then the relationship of underinvestment, overinvestment, and liquidity elaborated in the previous section is preserved also under this more general setup. Moreover, we can now study the interaction of liquidity and capital structure choice, as well as the trade-off between the value of tax benefits and the resulting agency costs upon debt issuance.

### 5.1 Debt Issuance

We first present the sequence of the debt issuance process. It takes place before state  $o$  is reached, and it is visualized by a dotted arrow in Figure 1. Consider a financially unconstrained entrepreneur who owns the investment option worth  $F^e(V)$  and follows the all-equity exercise policy. Preparing for debt issuance, the entrepreneur first provides some amount of liquid funds  $X$ . Then the unlevered firm's balance is given by (4).

In the next step, debt with a coupon  $C$  is issued. The prospective bondholders provide a payment of  $D^o(V)$  to the entrepreneur, where they correctly anticipate the value of their position in the firm after debt issuance. Upon debt issuance, the levered firm's balance changes to (3). The entrepreneur's optimization problem is to choose the liquid funds  $X$  and the capital structure  $C$  that maximize his gain upon debt issuance:

$$\max_{(X,C)} E^o(V) + D^o(V) - E^e(V) = F^o(V) - F^e(V) + T^o(V). \quad (24)$$

The left-hand side expresses that the entrepreneur receives a value of  $E^o + D^o$ , but he has to give up  $E^e$ . This also illustrates that at this point, he is still interested in maximizing the total value of equity and debt. The right-hand side allows the interpretation that he trades off the loss in investment option value  $F^o(V) - F^e(V)$  due to a suboptimal exercise policy against the tax benefits of debt  $T^o(V)$ .

## 5.2 Agency Costs

At first it might seem reasonable to define the agency costs of debt similar to Section 4 by comparing the firm value after debt issuance to that of an all-equity firm. This would correspond to the gain upon debt issuance given by (24). However, now an all-equity firm can be made better off by issuing debt due to tax benefits. In particular, a firm that can credibly commit to the first-best investment policy will in general increase its value by issuing debt, although its value was not affected by the choice of  $(X, C)$  for  $\tau = 0$ . Therefore it is no longer sufficient to compare investment option values before and after debt issuance, as was reasonable in (23). Rather, we have to take into account on the one hand under- and overinvestment costs driving the option value below that of an all-equity firm, and on the other hand costs due to foregone tax benefits. Our new definition of agency costs for  $\tau \geq 0$  is

$$AC_{\tau \geq 0} = [F_{(X_1, C_1)}^{o1}(V) - F_{(X_2, C_2)}^{o2}(V)] + [T_{(X_1, C_1)}^{o1}(V) - T_{(X_2, C_2)}^{o2}(V)] \quad (25)$$

The new superscript  $o1$  denotes values given that the first-best investment policy is followed after debt issuance, which means that when deriving the first-best investment threshold  $V_I^{o1}$  and bankruptcy threshold  $V_B^{o1}$ , the new first-order condition

$$E_V^{o1}(V_I^{o1}) + D_V^{o1}(V_I^{o1}) = E_V^i(V_I^{o1}) + D_V^i(V_I^{o1})$$

has to be used. In contrast, condition (20), i.e.

$$E_V^{o2}(V_I^{o2}) = E_V^i(V_I^{o2}),$$

is still used for the second-best investment policy (superscript  $o2$ , corresponds to  $o$  in the  $\tau = 0$  case). The bankruptcy thresholds will in general also be different, i.e.  $V_B^{o1} \neq V_B^{o2}$ , although they are both chosen in stockholders' interest. That means that condition (21) can now be stated for the first- and second best cases, respectively, as

$$E_V^{o1}(V_B^{o1}) = 0 \quad \text{and} \quad E_V^{o2}(V_B^{o2}) = 0.$$

Even in the first-best case, when it is possible to credibly commit to the first-best investment policy before debt issuance, the default threshold is not allowed to be chosen beforehand. Otherwise it would be ex-ante optimal to choose  $V_B^{o1} = V_B^{o2} = 0$ . In that case, the firm would never default, which would indeed maximize the value of tax benefits in our modelling framework, however would no longer represent the characteristics of a risky-debt financed firm. Only for the special case that the stockholders' net payment given by (2) is negative, it is also ex-post optimal from the stockholders' perspective to choose  $V_B^{o1} = V_B^{o2} = 0$ . Note that even in that case, a default threshold of  $V_B^i > 0$  after investment will be ex-post optimal.

The subscripts in (25) point out that if the entrepreneur can credibly commit to the first-best investment policy before debt issuance, he does not only choose different investment and bankruptcy thresholds. He also chooses a combination of liquid funds and capital structure  $(X_1, C_1)$  that will in general be different from the combination  $(X_2, C_2)$  that he chooses for the second-best investment policy. Note that for  $\tau = 0$ , the second bracket in (25) disappears and  $F_{(X_1, C_1)}^{o1}(V)$  corresponds to  $F^e(V)$ , so we end up again at definition (23).

### 5.3 Numerical Example and Discussion

In a numerical example, we show the  $(X, C)$  combinations actually chosen in the first-best and second-best cases, as well as the value effect of debt issuance and the resulting agency costs of debt. Since option values and tax benefits are connected to firm values by (3) and (4), we can rewrite (25) as the difference in gains upon debt issuance:

$$\begin{aligned} AC_{\tau \geq 0} &= [E_{(X_1, C_1)}^{o1}(V) + D_{(X_1, C_1)}^{o1}(V) - E_{(X_1)}^e(V)] \\ &\quad - [E_{(X_2, C_2)}^{o2}(V) + D_{(X_2, C_2)}^{o2}(V) - E_{(X_2)}^e(V)] \end{aligned} \quad (26)$$

Each of the square brackets represents the gain upon debt issuance relative to the all-equity firm for the first-best and second-best cases, respectively. The value of the first bracket (first-best case) as a function of the  $(X_1, C_1)$  combination chosen by the firm is shown in the first graph of Figure 6, while in the second graph the value of the second bracket (second-best case) is shown as a function of  $(X_2, C_2)$ . Parameter values are again given in Table 1. In contrast to Section 4, we now use a positive tax rate of  $\tau = 10\%$ , and the firm chooses the optimal levels of both liquidity and debt endogenously.

For a firm that follows the first-best investment policy, it can be seen in the first graph of Figure 6 that the maximum gain by debt issuance is reached for the maximum level of liquid funds,  $X_1^* = I = 100$ . While the liquid funds are not needed in order to implement a certain investment policy, they serve as a collateral and reduce the probability of default, and therefore the tax benefits can be enjoyed over a longer expected period of time. Since collateral causes no direct costs in our model, it is indeed optimal to use the maximum level of liquid funds. An interior level of debt (coupon  $C_1^* = 30.02$ ) is chosen in order to achieve the maximum value of tax benefits, which is a well-known result from dynamic trade-off models.<sup>15</sup>

The second graph of Figure 6 shows the change in firm value upon debt issuance given that the firm follows the second-best investment policy. Still, an interior level of debt (coupon  $C_2^* = 31.02$ ) close to that of the first-best case is chosen. However, as a result of

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<sup>15</sup>See e.g. Leland (1994).

the trade-off between tax benefits and agency costs of debt, the maximum gain by debt issuance is now reached for an interior level of liquid funds  $X_2^* = 72 < I$ .

As shown for the case without tax benefits in Figure 5, the firm needs a certain level of liquid funds in order to implement a second-best investment policy that mitigates conflicts of interest between stockholders and bondholders and thus agency costs of under- and overinvestment. Therefore it makes sense that the level of liquid funds  $X_2^*$  is below the maximum level of liquid funds,  $X_1^* = I = 100$ , which is chosen by a firm that can credibly commit to the first-best investment policy. However, while minimizing the loss  $F^e(V) - F^o(V)$  was the only effect in Section 4, now the value of tax benefits,  $T^o(V)$ , has to be considered as well. Therefore the resulting  $X_2^*$  is on the other hand above the level of liquid funds  $X_0^*$  that the firm chooses for a similar leverage when there are no tax benefits, until the marginal gain in tax benefits equals the marginal agency costs.

The resulting maximum gains upon debt issuance are 14.23 in the first-best case, and 13.49 in the second-best case, respectively. We can then use (26) and calculate the actual agency costs of debt as the difference of the maximum gains upon debt issuance in the first- and second-best cases. In our numerical example this corresponds to a value of  $14.23 - 13.49 = 0.74$ . This means that the gain upon debt issuance in the second-best case is about 5% smaller than in the first-best case.

One could argue that the effect of the agency costs of debt is rather small. However, this is most importantly due to the fact that the firm is significantly adjusting its liquid fund holdings in the second-best case. If the firm kept the maximum liquid fund holdings in the second-best case (i.e.  $X_2 = 100$ ), there would be a maximum gain upon debt issuance of only 10.51.<sup>16</sup> Then the agency costs of debt would amount to  $14.23 - 10.51 = 3.72$ . This means that the gain upon debt issuance in the second-best case would be more than 26% smaller than in the first-best case. Overall, our main conclusion from the analysis without tax benefits therefore remains valid: The firm's liquid funds are an important factor determining the agency costs of debt, and they have to be chosen carefully in order to avoid a significant loss in value.

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<sup>16</sup> In this case, the optimal second-best coupon would be  $C_2 = 31.69$ .

In order to ensure the stability of our results, we carried out a comparative statics analysis. We could confirm that the basic qualitative structure remains unchanged for a wide range of parameter values: While it is optimal to choose the maximum level of liquid funds in the first-best case, there is a significantly lower level of liquid funds in the second-best case. On the one hand, cash holdings help to increase tax benefits. However, there would be a tremendous overinvestment problem if the firm stuck to the first-best policy of holding the maximum level of liquid funds although it could not commit to the first-best investment policy.

## 6 Implications and Related Work

The empirically testable implications of our model for levered firms can be summarized as follows:

I. For a given interior level of liquidity, investment thresholds are U-shaped in leverage.

There is overinvestment for low leverage and underinvestment for high leverage.

II. For a given positive leverage, investment thresholds are decreasing in liquidity.

There is underinvestment for low liquidity and overinvestment for high liquidity.

III. An interior level of liquidity and leverage is optimal.

It trades off agency costs of debt (under- and overinvestment) and tax benefits.

Most existing models concentrate on either under- or overinvestment. Among those, the more frequently examined is the underinvestment effect of Myers (1977). For example, a recent paper by Moyen (2007) deals with that issue with respect to debt maturity.

In order to model the overinvestment effect, the literature usually introduces quite specific financing contracts. One recent example is the article by Mauer and Sarkar (2005). The authors assume that an otherwise all-equity firm issues a conditional debt contract, which means that the coupon payment required for a given lending amount is already specified

ex-ante, while the firm receives the lending amount at the time of investment. However, the authors do not allow the firm to choose between several alternative pre-specified debt contracts upon investment, but the exact level of leverage has to be specified ex-ante. Overinvestment can also be observed if a levered firm's investment is partly financed using a new debt issue upon investment. This has been shown e.g. by Nippel (1996) and is further examined in two recent papers by Lyandres and Zhdanov (2006) and Sarkar (2007). Lyandres and Zhdanov (2006) neutralize the underinvestment effect using a not necessarily optimal financing contract and focus on overinvestment incentives due to the threat of costly default. In contrast, Sarkar (2007) derives the optimal expansion debt financing contracts from a first-best and second-best perspective.

There is one article by Childs, Mauer, and Ott (2005) that is particularly related to our paper: In their approach, investment is also partly financed using existing assets, which allows to capture both under- or overinvestment in one model. Like Moyen (2007), the authors focus on the implications for the firm's debt maturity structure. They develop a rich model, but in their analysis they have to confine themselves to the comparison of two specific parametrizations representing either under- or overinvestment. It remains unclear which of the two firms' characteristics are most relevant for the fundamentally different investment policies in either situation. Moreover, the two investment environments are exogenous for the firms. There is no endogenous decision on the asset side that would allow to influence if the firms actually face under- or overinvestment.

In contrast, this decision is in the focus of our paper. We propose the firm's liquid funds as an easily adjustable parameter on the asset side, which allows the firm to actively influence the trade-off between under- and overinvestment.

Our Implication I is in fact the result of an interior level of liquidity. Whenever the investment situation is too close to a pure asset-expansion problem for low liquid funds, or a pure asset-substitution problem for high liquid funds, then there is monotonicity of the investment threshold in leverage. Since the above mentioned studies are examining these boundary cases, they do not find investment thresholds that are U-shaped in leverage.

Our Implications II and III are both directly related to the level of liquid funds and its meaning for under- and overinvestment. Since the mentioned studies do not consider the firm's liquid funds explicitly, they do not find these implications. Most related to our Implication III is the finding of Sarkar (2007): In the case where a levered firm's investment is partly financed using a new debt issue, the second-best policy is also the result of a trade-off between agency costs of under- and overinvestment on the one hand, and tax benefits and bankruptcy costs on the other hand.

## 7 Conclusion

In this work we have introduced the firm's level of internal liquid funds as the key factor that determines the effect of existing debt on investment. We have shown that the distortions in investment policy resulting from stockholder-bondholder agency conflicts are increasing in the firm's leverage. This holds for the extreme cases of a firm holding no liquid funds, resulting in underinvestment, and enough liquid funds to finance the investment project, inducing overinvestment, respectively.

However, the two effects interact for any interior level of liquid funds, inducing an investment threshold that is U-shaped in leverage: Overinvestment due to asset substitution and risk-shifting incentives is more important for small amounts of debt. For higher leverage, the reluctance to provide additional value to the bondholders and thus the underinvestment argument becomes predominant. This is a central and new finding of our paper.

Note that we do not assume that the firm is financially constrained. In our case, underinvestment is not due to the stockholders' difficulties to raise the investment amount on external capital markets. Rather, they decide willingly that they do not want to provide the amount since it also benefits the bondholders. In contrast, for a sufficiently high level of liquidity being held within the firm, the benefits to the stockholders that arise from asset substitution outweigh the cost from asset expansion, and overinvestment takes place. Regarding the firm's liquid funds as the decision variable for a given leverage level, we conclude that investment thresholds are decreasing in liquidity for levered firms. There is

an optimal level of liquidity for which there is a trade-off between these two effects, and the ex-ante optimal investment policy of an all-equity firm can be implemented. Even if the latter is not directly enforceable, we can therefore eliminate the agency costs of debt. For any other firm liquidity, the overall level of agency costs is determined by the degree of deviation from the optimal level of liquidity.

Increasing project risk leads to earlier investment. This holds both in cases where there is overinvestment and underinvestment, respectively. Consequently, while underinvestment can be mitigated for increasing project risk, the overinvestment problem becomes more severe when the shareholders have more incentives for risk-shifting. Again, the ex-ante optimal investment policy can still be implemented by choosing an appropriate, i.e. a lower level of liquidity for increasing project risk.

In a second step of our analysis, we have examined the interaction of liquidity and capital structure choice given that a tax advantage of debt makes leverage favorable. In that case, a firm that can credibly commit to a certain investment policy has a motivation to issue debt, and it will use the maximum level of liquid funds as a collateral for a higher value of tax benefits. In contrast, if the investment policy cannot be bound by contracts, the firm chooses an interior level of liquid funds in order to implement an investment policy that both mitigates conflicts of interest between stockholders and bondholders on investment policy and helps to increase the value of tax benefits.

Overall, we have shed light on the key factor that determines the actual effect of existing debt, namely the firm's level of liquidity, and thus we have provided some answers to the introducing question which conditions lead to under- or overinvestment of levered firms.

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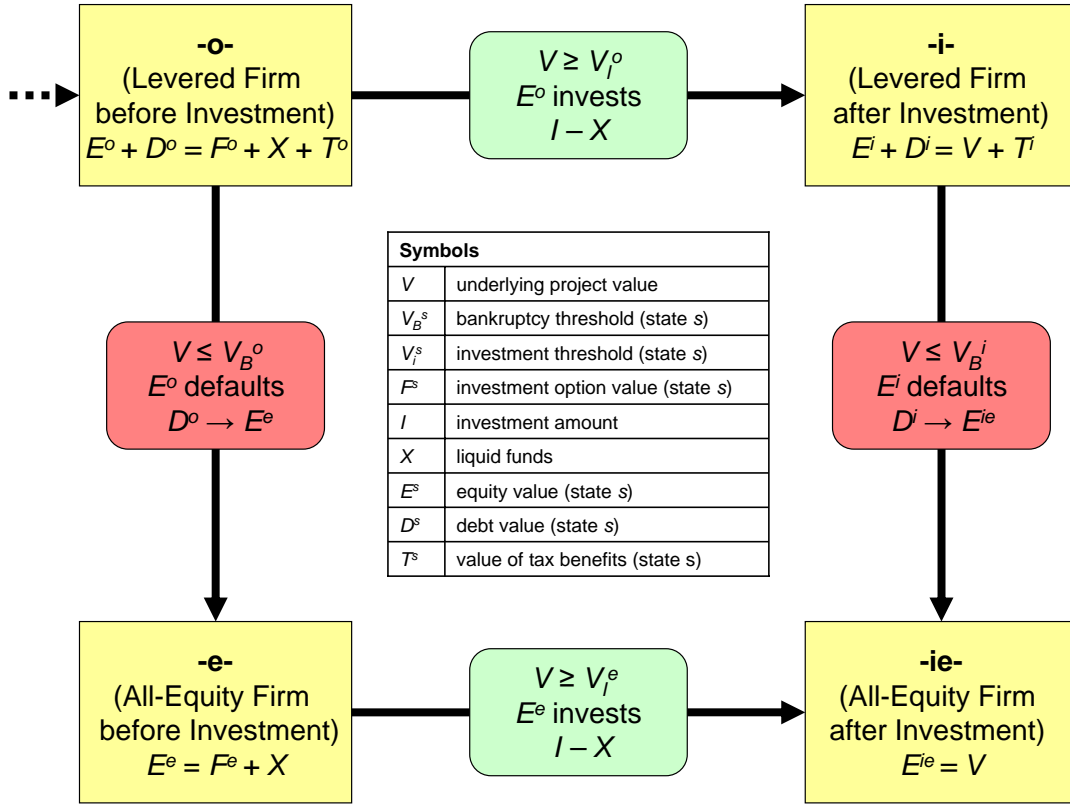


Figure 1: Transition of States.

This figure shows the possible state transitions of the firm contingent on the evolution of the project value  $V$ . Starting in state  $o$ , the firm will invest as soon as  $V_I^o$  is exceeded. Then the stockholders have to provide the amount  $I - X$ . In contrast, if the project value first falls below  $V_B^o$ , the firm will default. After default (state  $e$ ), the former bondholders will be the owners of an all-equity firm, and they decide when to invest by providing the amount  $I - X$  as soon as  $V_I^e$  is exceeded. On the other hand after investment (state  $i$ ), default is still possible, and the firm will default if the project value falls below  $V_B^i$ . In either of the latter states, the next transition leads to state  $ie$  in which both investment and default have already taken place. The dotted arrow into state  $o$  represents the preceding debt issuance decision of an all-equity firm.

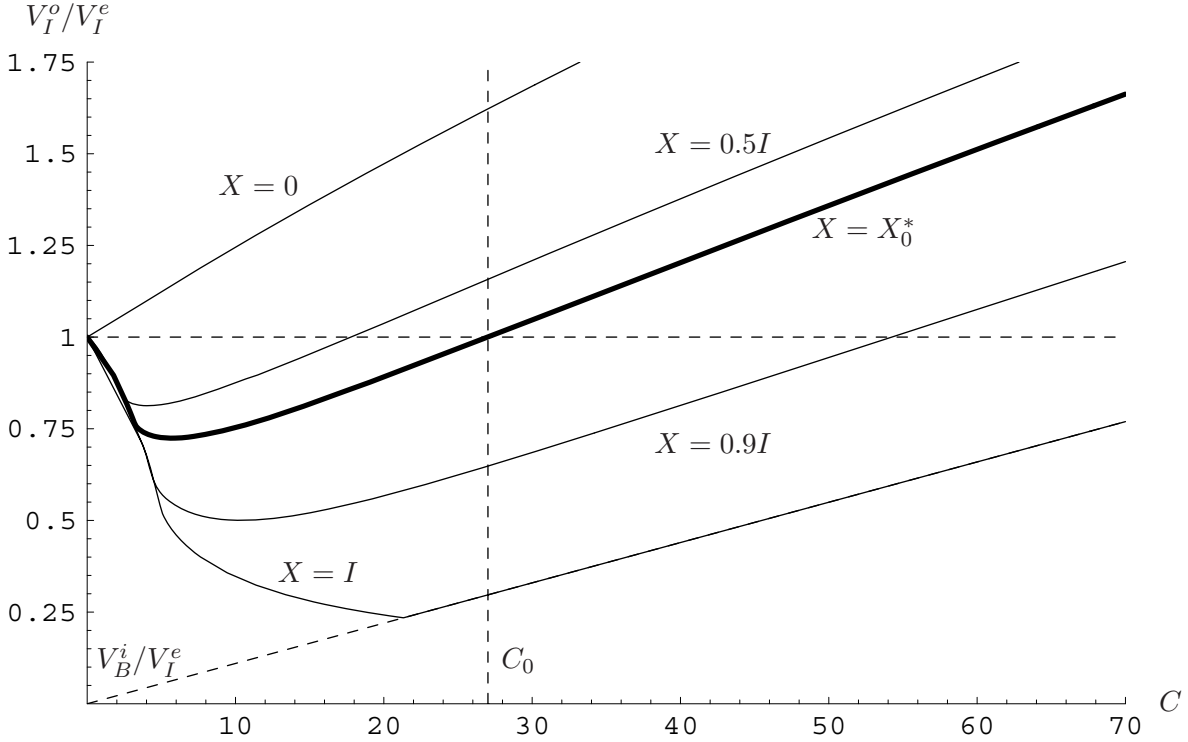


Figure 2: Levered Firm's Investment Threshold as a Function of Leverage.

The levered firm's investment threshold  $V_I^o$ , normalized by the investment threshold of the unlevered firm  $V_I^e$ , is shown for different levels of liquid funds (solid lines) as a function of the firm's leverage measured by the debt coupon  $C$ . The firm follows the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. Also shown is the investment threshold of the unlevered firm, which is normalized to 1 and independent of  $C$ , and the bankruptcy threshold after investment  $V_B^i/V_I^e$ . The coupon level  $C_0$  will be fixed in Figure 3. Parameter values are given in Table 1, taxes are not considered ( $\tau = 0$ ).

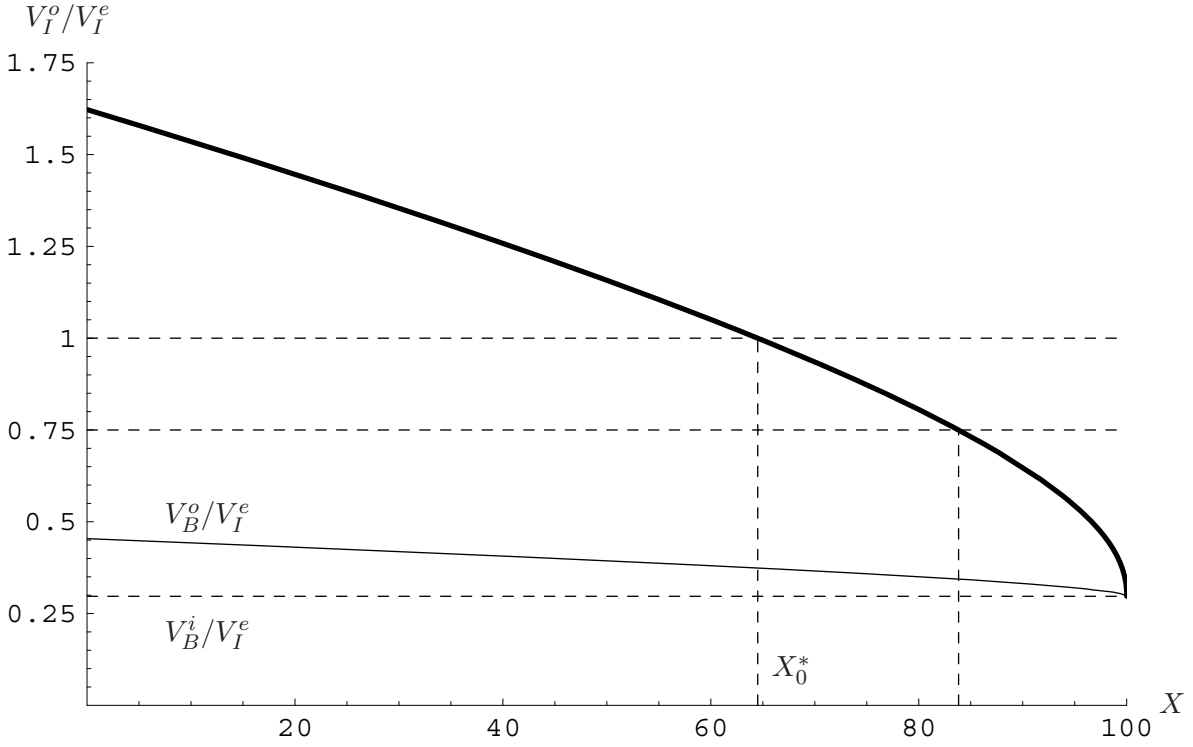


Figure 3: Levered Firm's Investment Threshold as a Function of Liquid Funds. The levered firm's investment threshold  $V_I^o$  (solid line), normalized by the investment threshold of the unlevered firm  $V_I^e$ , is shown as a function of the firm's liquid funds  $X$  (for  $C = C_0$ ), given that the firm follows the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. Also shown are the investment threshold of the unlevered firm which is normalized to 1 and the  $V_0/V_I^e = 0.75$  line indicating the level later used as the initial project value, as well as the bankruptcy thresholds before and after investment,  $V_B^o$  and  $V_B^i$ , respectively. Parameter values are given in Table 1, taxes are not considered ( $\tau = 0$ ).

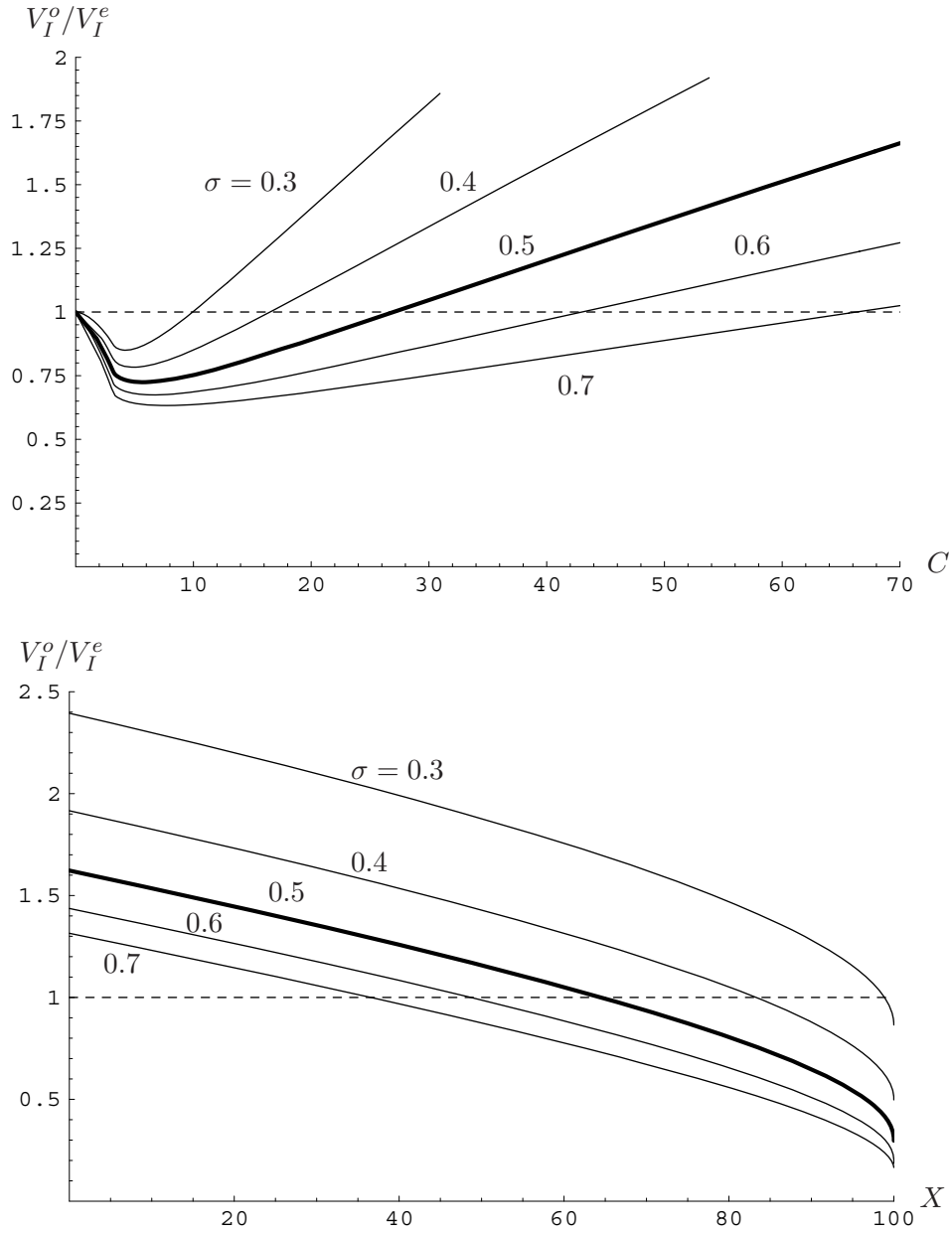


Figure 4: Levered Firm's Investment Threshold for Varying Project Value Volatility. The levered firm's investment threshold  $V_I^o$  (solid lines), normalized by the investment threshold of the unlevered firm  $V_I^e$ , is shown as a function of the firm's leverage measured by the debt coupon  $C$  (for  $X = X_0^*$ ), and the firm's liquid funds  $X$  (for  $C = C_0$ ), respectively, given that the firm follows the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. Also shown is the investment threshold of the unlevered firm which is normalized to 1. Parameter values are given in Table 1, taxes are not considered ( $\tau = 0$ ).

Change in Firm Value =  $-AC$

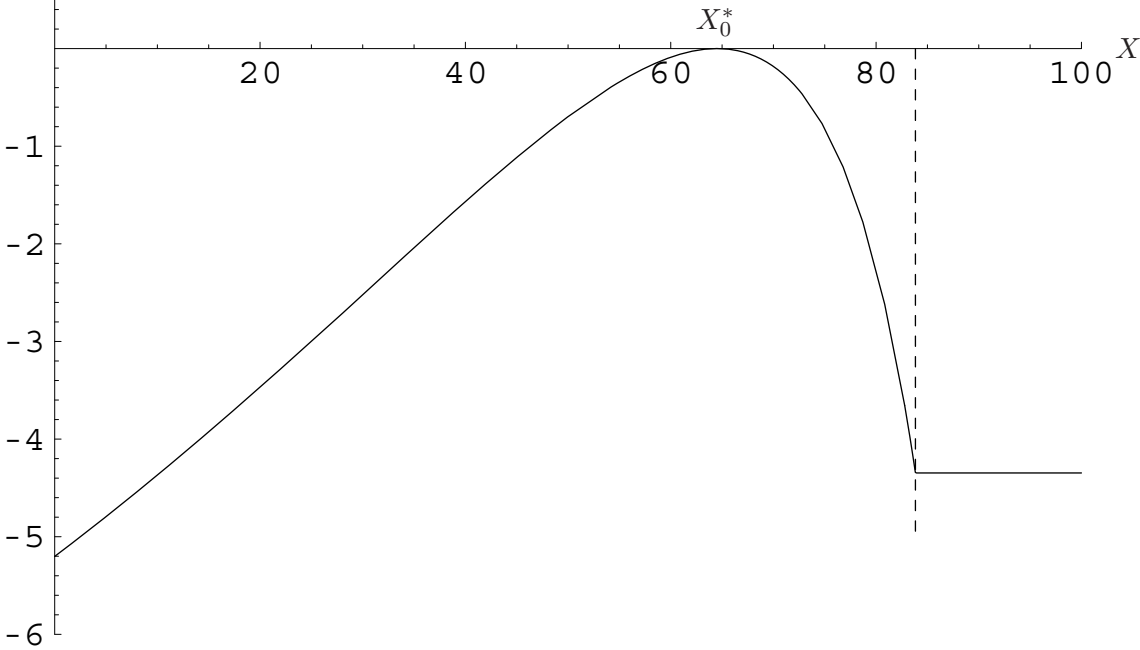


Figure 5: Agency Costs of Debt.

The agency costs of debt, measured by the (negative) change in firm value upon debt issuance, are shown as a function of the firm's liquid funds  $X$ , given that the firm follows the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. Parameter values are given in Table 1, the initial project value considered is  $V_0 = 0.75V_I^e = 319.92$ , and taxes are not considered ( $\tau = 0$ ).

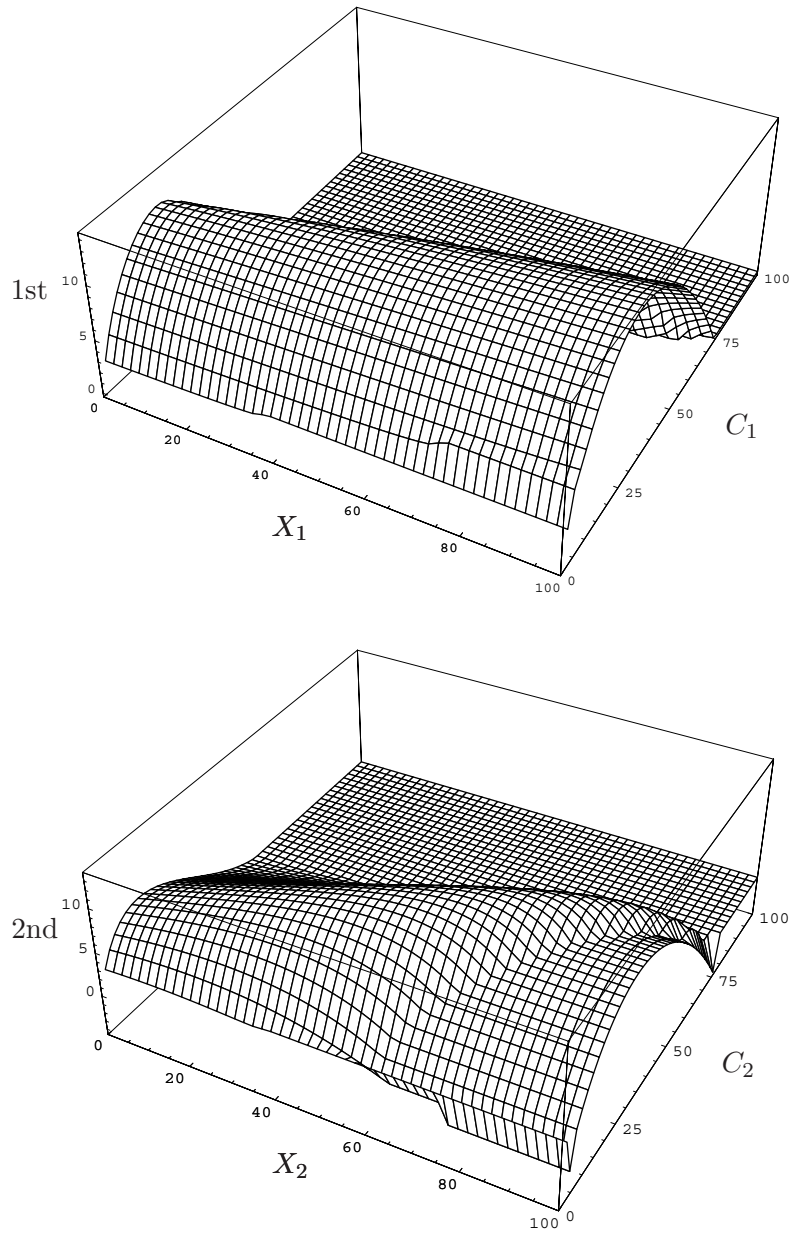


Figure 6: Optimal Liquidity and Capital Structure.

The change in firm value upon debt issuance is shown as a function of the firm's liquid funds  $X_{1/2}$  and debt coupon rate  $C_{1/2}$ , given that the firm follows the first-best investment policy, i.e. the policy that is value-maximizing before debt issuance (first graph), or the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance (second graph), respectively. Parameter values are given in Table 1, the initial project value considered is  $V_0 = 0.75V_f^e = 319.92$ , and the tax rate is  $\tau = 10\%$ .