Leaders, Followers and Risk Dynamics in Industry Equilibrium*

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This version: November 20, 2007

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JEL Classification: C23, C35.

Keywords: Growth options and industry risk, asset pricing and investment decisions, risk dynamics in oligopolistic industries

1 Introduction

Traditional asset pricing largely ignores the influence of industry structure and competitors' reactions when analyzing the risk dynamics of a single firm. Nevertheless it seems obvious that investment decisions in an oligopolistic market, that cause the existing capacity of rivals to change, also determine the cash flows earned by the firms in the industry, and hence influence the risk dynamics. Therefore it seems natural to link industry structure with asset returns and risk.

The recent literature on corporate investment decisions using a real options framework helps to bridge the gap between asset pricing and industrial organizations. Grenadier (2002) introduces a generalization of McDonald and Siegel (1986) in which symmetric oligopolists face the option to expand capacity in an industry where demand is stochastic. Assuming that firms operate in a homogenous product market he derives a symmetric equilibrium with strategic exercise of options and shows that the value of the option to wait diminishes as the industry becomes more competitive (i.e. the number of firms increases). Aguerrevere (2003) takes up the model of Grenadier (2002) but allows for operating flexibility, i.e. firms need not produce with full capacity. He shows that firms might find it optimal to expand capacity even if current output levels do not fully use existing capacity and that endogenous output price fluctuations are increasing with the number of competitors. While both these papers analyze the link between industry competition and capacity expansion, they do not look at the risk implications of investment strategies.

Aguerrevere (2005) is the first that relates industry structure and risk dynamics in a model with N symmetric firms. He introduces two alternative settings. One in which the symmetric firms face fixed capacities that can only be changed by exercising a growth option, the other one in which firms have operating flexibility. He finds that in case of operating flexibility industry risk is higher the more competitive the industry is. Industry risks are an immediate consequence of operating leverage and irreversibility. While this is an interesting result that has empirical support found in the paper by Hou and Robinson (2005), it is not clear which one of the two effects dominates. To explore this, an approach is required that looks at asymmetric firms in an industry setup.

There are several papers in the real options literature that study corporate investment decisions in an oligopolistic industry when firms face options to expand. Boyer, Lassere, Mariotti and Moreaux (2004) formulate a model with asymmetric firms that strategically exercise growth options. They concentrate on preemptive effects of capacity expansion and do not study asset price dynamics. Pawlina and Kort (2002) study an asymmetric duopoly model with a single option for each firm and look at different exercise equilibria.

In this paper we study corporate investment decisions of asymmetric firms that compete in a duopolistic output market and analyze the risk dynamics in such a setup. In particular, we look at a market in which two firms supply a homogenous product. Demand in the market is stochastic and follows a Geometric Brownian Motion (GBM). Each firm in the industry starts out at a given capacity level with fixed costs but holds an option either to shrink or to expand capacity depending on the level of demand. After expansion (contraction) each firm operates with a higher (lower) level of capacity, output and fixed costs. Option exercise is costly. In case of an expansion investment costs include both adjustment costs and the price of the investment and in case of contraction firms get paid a salvage value for the downsized units.[1]

We assume that investment costs and salvage values are asymmetric so that there is a high cost and a low cost firm and a firm with a high and a low salvage value. In both cases option exercise is sequential with the low cost (high salvage value) firm expanding first (being the leader) and the high cost (low salvage value) firm expanding second (being the follower). This sequential exercise gives rise to three different industry stages: a juvenile industry in which neither firm exercised any of the options, an adolescent industry in which the leader exercised his option but the follower did not, and a mature industry in which both firms have exercised their options.

We find that the risk dynamics of the firms are driven by both operating leverage and irreversibility, two effects that have been recognized the existing investment literature (see for example Carlson, Fisher, and Giammarino (2004)). In addition we identify a strategic (industry) risk factor that arises

[1]The introduction of both an expansion and contraction option is motivated by the assumption that firms have to produce at capacity levels. Allowing for both types of options can be seen as a substitute for operating flexibility.

because of imperfect product market. It turns out, for example, that in an adolescent industry where a leader has expanded but a follower has not, the leader's risk is reduced by the follower's potential. Hence, relative to a monopoly situation, the existence of a competitive rival results in a reduction of risk.

The intuition for this result comes from what we refer to as a hedging effect. If both firms were at full capacity with no ability to expand or contract, any demand shock would translate directly into a price shock. Now consider the situation where one firm has expanded but the other has not. There is some demand shock that would induce the follower to exercise their expansion option. The increase in capacity will dampen the demand shock, thereby reducing risk. Similarly, the ability to contract will off set negative demand shocks.

As pointed out above, the risk reducing effect in any of the two cases (expansion and contraction) is driven by a hedging argument. The adjustment of industry output as a consequence of option exercise of the rival causes prices to change less widely so that firms face a risk reducing effect. As it turns out the industry risk effect has opposite risk implications in an expanding industry and same risk implications in a shrinking industry. This prediction has important implications for empirical analysis. If we add industry factors to firm's own risk characteristics we have to expect both, same and opposite risk implications. Our empirical analysis shows that industry characteristics influence firm returns.[2]

Our findings are also able to disentangle the two driving forces behind risk dynamics of individual firms. As many recent papers on corporate investment decisions and asset price dynamics, such as Gomes, Kogan and Zhang (2003), Zhang (2005), and Cooper (2006), point out, the risk dynamics are governed by operating leverage and the degree of irreversibility. While both effects are important, it is not clear how they can be separated and identified by observable variables. Our approach documents that operating leverage must be associated with the firm's book to market, while irreversibility can be captured by industry book to market. This implies a set of new testable hypotheses that can shed new light in the empirical

[2]Our findings provide an interesting contrast to Novy-Marx (2007). Based on a symmetric equilibrium, he predicts and finds strong within industry effects and weak between industry effects.

asset pricing literature. It is important to point out, that only a framework with asymmetric firms is able to separate these two effects. In a symmetric equilibrium both effects occur simultaneously and therefore cannot be isolated.

Our paper is organized as follows. In Section 2 we present the model. Section 3 presents our main results through a series of special cases. First we consider the case of a flexible firm that can expand or contract and a rival firm that is unable to do either. The inflexible firm shows most clearly the industry effects, i.e. the fact that even without growth options, the risk faced by a producing firm responds to the growth options held by rivals. We then look at the consequences of two firms having growth options, followed by an examination of two firms who have contraction options. Section 4 presents empirical evidence that clearly show that firm returns are related to both industry and own firm characteristics. Section 5 concludes the paper.

2 Firms' Growth in an Asymmetric Duopoly

In this section we present the model in which two strategically interacting firms invest in productive capacities and compete in output levels. We first present the structure of the model and then derive the general valuation equation for each firm. Firm values are driven by assets in place including the level of fixed costs and option values to either expand or contract existing capacity.

2.1 Industry Demand, Production Technologies, and Capital Accumulation

We consider an industry in which two firms produce a homogenous product. Denote the industry output rate at instant t by $Q_t = Q_t^1 + Q_t^2$, where Q_t^1 and Q_t^2 are the output levels of firm one and two, respectively. The industry output price is determined by the iso-elastic inverse demand curve

$$P_t = X_t Q_t^{\gamma-1}, \tag{1}$$

where $0 < \gamma < 1$, and X_t is an exogenous state variable that represents the level of industry-wide demand. The dynamics of X_t are specified by

$$dX_t = gX_t dt + \sigma X_t dW_t, \tag{2}$$

where dW_t is the increment of a Wiener process, g is the constant drift, and σ^2 the constant variance.

Firm i produces output at time t using installed capital K_t^i where $i \in \{1,2\}$. Any capital level K^i is associated with a maximum output level $Q^i\left(K^i\right)$, which implies that $Q_t^i \leq Q^i\left(K_t^i\right)$. For simplicity, capital levels take one of three discrete values: $K_t^i \in \{\kappa_0, \kappa_1, \kappa_2\}$, where $\kappa_0 < \kappa_1 < \kappa_2$. Costs of production for firm i at date t are given by the increasing function $F_t^i = f\left(K_t^i\right)$. This cost structure emphasizes the impact of operating leverage, since total expenditures depend only on the installed capital level K^i (as with, for example, overhead). Given the three possible capital levels, there are also three possible levels of operating costs: $F_t^i \in \{f_0, f_1, f_2\}$, where $f_0 < f_1 < f_2$.

To move from one capital state to another, the firm may incur costs or generate revenues, either from buying or selling the productive asset, or from pure adjustment costs. To capture this idea in a general way, we specify for each firm a matrix of discrete transition costs:

$$\Lambda^i \equiv \begin{bmatrix} 0 & \lambda_{01}^i & \lambda_{02}^i \\ \lambda_{10}^i & 0 & \lambda_{12}^i \\ \lambda_{20}^i & \lambda_{21}^i & 0 \end{bmatrix}. \tag{3}$$

The instantaneously incurred lump-sum cost for firm i to move from capital level κ_m to κ_n is given by λ_{mn}^i. The only source of heterogeneity between firms in our model is the difference between transition costs, i.e., $\Lambda^1 \neq \Lambda^2$. We assume as an initial condition that at date zero, each firm is endowed with $K_0^i = \kappa_1$ units of capital.

We finally define indicator variables D_t^{imn} that take the value one at the instant when firm i switches from capital level κ_m to κ_n, and zero elsewhere.[3] For convenience, we denote by D_t^i the matrix of investment decisions D_t^{imn}.

[3]Formally, we can define $D_t^{im} \equiv d1_{\{K_t^i=\kappa^m\}}$, which represents the decision of firm i to enter or leave state m. The variable D_t^{im} thus takes the value 1 at the instant firm i enters state m and takes the value -1 at the instant the firm leaves state m. It takes the value 0 everywhere else. The equation $D_t^{imn} \equiv \left|D^{im}\right| D^{in} \left[\left(D^{in} - D^{im} + 2\right)/4\right]$ then gives a family of indicator variables with the desired properties.

2.2 Output, Investment Strategies, and Equilibrium

The economy described above is a dynamic game between firms 1 and 2. At each instant, the managers of the two firms choose output rates Q_t^i and make investment decisions D_t^i knowing the complete history of the game denoted by $\Phi_t = \left(\left[Q_s^1, Q_s^2, K_s^1, K_s^2\right]_{s<t}, [X_s]_{s\leq t} \right)$, which is common to both managers.

We define the payoff to firm i as the present value of the expected discounted future cash flows. The cash flows at time t derive either from revenues in excess of fixed costs $\pi_t^i \equiv P_t Q_t^i - F_t^i$ or from lumpy investment costs related to the decision D_t^i. We assume the absence of agency conflicts, so that manager i maximizes the value function

$$V_t^i \equiv E_t \int_t^{\infty} e^{-r(s-t)} \frac{M_{t+s}}{M_t} \left[\pi_{t+s}^i ds + \mathbf{1}' \left(D_{t+s}^i * \Lambda^i\right) \mathbf{1}\right], \tag{4}$$

where $M_0 = 1$ and $dM_t = \frac{\mu - r}{\sigma} M_t dW_t$ represent the pricing kernel, $\mathbf{1}' = [1,1,1]$, and $*$ represents element-by-element multiplication.

Given the Markov structure of this environment, it is natural to restrict attention to Markov strategies. Manager i can then take actions Q_t^i and D_t^i that depend only on the most recently observed values of the payoff relevant state variables X_t and $K_{t-} \equiv \left(K_{t-}^1, K_{t-}^2\right)$, where $K_{t-}^i \equiv \lim_{s \uparrow t} K_s^i$. A pure strategy Markov-perfect equilibrium (MPE) of the game is a pair, $i = 1,2$, of vector-valued functions $\left[Q^i, D^i\right](K_{t-}, X_t)$. In an equilibrium strategy pair, each must maximize the value function (4) in every state (K_{t-}, X_t), conditional on the equilibrium strategy of the rival.

It is straightforward to show that any MPE must have quantity choices equal to static Cournot equilibrium output levels. In our setting, value maximizing firms always endogenously produce at their capacity limits due to zero marginal costs and sufficiently low price elasticity. Hence, any MPE strategy requires $Q_t^i = Q^i\left(K_t^i\right)$. The instantaneous profit functions $\pi_t^i = \pi^i(K_t, X_t) = X_t \left[Q^1\left(K_t^1\right) + Q^2\left(K_t^2\right)\right]^{\gamma-1} Q^i\left(K_t^i\right) - F_t^i$ are thus fully determined by the current capital levels K_t^1 and K_t^2 and the value of the state variable X_t.

To aid future exposition, it is convenient to define the capital dependent *revenue factors*

$$R^1_{mn} \equiv \left[Q^1(\kappa_m) + Q^2(\kappa_n)\right]^{\gamma-1} Q^1(\kappa_m),$$
$$R^2_{mn} \equiv \left[Q^1(\kappa_m) + Q^2(\kappa_n)\right]^{\gamma-1} Q^2(\kappa_n),$$

where $m, n \in \{0, 1, 2\}$ index the capital levels of firms 1 and 2, respectively. We can then conveniently write the profit function of each individual firm i as $\pi^i\left(K^1_t = \kappa_m, K^2_t = \kappa_n, X_t\right) = X_t R^i_{mn} - F^i_t$.

Given the simplification of the instantaneous output choices Q^i_t, we can henceforth focus attention on the dynamic game of option exercise involving the capital levels K^i_t and the investment decisions D^i_t. Any Markov strategy can be summarized by a set of exercise boundaries that for each player i and each capital state K_{t-} specify regions of the state variable X_t at which player i will change his capital level to a new state. We can use standard techniques of backward induction to derive MPE of the dynamic game.

In order to focus attention on the marginal impact of a firm's decisions on its own value, standard practice separates valuation equations into (1) the present value of the existing operations of the firm as a going-concern, and (2) the incremental value of any real options. Following this logic, in our settings we can consider the decomposition

$$V^i(K_t, X_t) = V^i_A(K_t, X_t) + V^i_F\left(K^i_t\right) + V^i_O(K_t, X_t), \tag{5}$$

where V^i_A denotes the value of revenues generated by assets-in-place,V^i_F is the present value of fixed costs, and V^i_O is the value of real options. In this equation, the first two components aggregate to going-concern value, and the last component reflects the value of current or future actions to modify operations.

For a monopolist, the impact of own-firm decisions on own-firm value and risk are fully reflected in decompositions such as (5). In a duopoly there are consequences of firm actions on rivals value and risk. In the next section we examine specific examples that allow a richer decomposition that identifies what we will refer to as 'own' and 'industry' effects.

3 Optimal Capacity Adjustment: Specific Examples

In this section we explore several special cases of our general model. In the first, we assume that one firm is flexible in that it has both an expansion and a contraction option, while the second firm is inflexible and is not able to adjust capacity at all. The inflexible firm in this example is a benchmark in showing clearly how actions by rivals impact value and risk independent of own actions.

In a second example both firms are flexible and are equipped with a single expansion option. Here we are able to characterize the equilibrium of the corresponding capacity adjustment game and explore the interactions of firms when both are flexible and able to increase their capacity levels.

The third example allows for a single contraction option of each firm. It turns out that this example has some distinguishing features compared to the expansion case. Again we derive the firm values and the risk dynamics arising in this scenario.

3.1 Industry Risk vs. Own Firm Risk

This subsection explores the simplest case of the general model developed in Section 2. Specifically, we assume that one rival is *flexible*, and begins with one option to either expand or contract, while the other rival is *inflexible* and has no ability to change its capital level. This allows us to add to the standard decomposition (5) by recognizing that in competitive environments (either oligopolistic or perfectly competitive) rival actions also have important impacts on firm value. In this paper, we focus on cross-firm valuation and risk effects that arise due to product market interactions. When product markets are shared, one firm's actions to either expand or contract output alter the output price dynamics of the industry, and hence contribute to valuation and risk dynamics of competitors.[4] We thus consider an extended decomposition in which both the value of the assets in place V_A^i and the option value V_O^i are related to actions taken by the firm and actions taken by the rival.

[4]Similar cross-firm real option effects could also be transmitted through factor input markets.

This scenario is useful because it allows us to isolate the two sources of real option risk that can occur in a real options duopoly. The flexible firm has risk that depends only on its own real option and leverage. The presence of an inflexible rival creates a permanent dampening of risk, but has no dynamic impact on the flexible firm risk loadings. As in the monopoly case explored by CFG (2004, 2006), the flexible firm thus has an own-option-leverage risk component but no independent source of dynamic industry risk.

By contrast, the inflexible firm offers the polar opposite case. The inflexible firm has no own-option-leverage component in its risk loadings, but nonetheless, it is exposed to dynamic risk due to the investment decisions of its rival. Thus, the inflexible firm stock price is exposed only to dynamic industry risk and has no independent source of own-option-leverage.

This example highlights that the presence of a rival is risk-reducing. Intuitively, when a competitor is near an option exercise boundary, its investment decisions act as a natural hedge against variations in the exogenous state variable. For example, when a rival is near an expansion boundary, good news about demand going up will be offset by the bad news that the competitor is closer to expanding, which will drive output prices down. A similar effect also hedges risk near competitor contraction boundaries. Future sections consider cases where both own-firm and rival-firm options are present, and we find that the presence of a rival may tend either to reinforce or to offset the own-firm effect depending on the circumstance.

As a result, we expand the decomposition in equation

To achieve a specification where one firm is flexible and the other inflexible, we set the capital adjustment costs to

$$\Lambda^1 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ S & 0 & -I \\ -\infty & -\infty & 0 \end{bmatrix} \qquad \Lambda^2 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix}, \qquad (6)$$

where $S, I > 0$. Firm 1, the flexible firm, thus begins at capital level κ_1 and has a single option to change capacity, either by expanding to κ_2 or contracting to κ_0. If it expands, it pays the investment cost I and if it contracts it receives the salvage value S. Once firm 1 either expands or contracts, it has no further options to change capacity. Firm 2 begins at capital level κ_1 and has no options to change capacity.

We now examine the exercise decision of firm 1 (the flexible firm). Taking the operating decisions of firm 2 as given, the flexible firm policy is described by a critical demand level X_E at which the firm will expand, and a demand level $X_C < X_E$ at which it will contract. The value of the flexible firm is summarized as our first result. Let us denote by $V^1_{AM}(K_t, X_t)$ the value of the assets in place of firm 1 and by $V^1_{OM}(K_t, X_t)$ the investment option value due only to the firm's own actions.

Proposition 3.1: The value of firm 1, the flexible firm, is the sum of the value of the assets in place net of the present value of fixed costs and the option value, i.e.,

$$\begin{aligned} V^1(K_t, X_t) &= \frac{R^1_{11} X_t}{\delta} - \frac{f_1}{r} + B^1_1 X_t^{\nu_1} + B^1_2 X_t^{\nu_2}, \\ &= V^1_{AM}(K_t, X_t) + V^1_F(K_t) + V^1_{OM}(K_t, X_t), \end{aligned} \tag{7}$$

where B^1_1 and B^1_2 are positive constants determined by the boundary conditions, and $\nu_1 > 1$ and $\nu_2 < 0$ are the solutions to the characteristic equation

$$\frac{1}{2}\sigma^2 \nu(\nu - 1) + (r - \delta)\nu - r = 0, \tag{8}$$

given by

$$\nu_{1,2} = \frac{1}{2} - \frac{r-\delta}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{9}$$

Proof: See Appendix A.
As in standard real option models firm value is given by the value of the assets in place adjusted for the fixed costs and the option value. While both values are only related to firm 1 own action they do reflect the initial capacity and output of both firms in the industry. Since firm 1 is the flexible firm the value of the assets in place and the option value is only driven by its own actions. There is no value arising from the competitors actions. As in Leahy (1993) the flexible firm acts like a myopic firm who's valuation is only driven by its own actions.

The value of the inflexible firm derives only from the assets in place since there are no options available. The value of the assets in place, however, reflect the fact that there will be revenue changes every time the flexible firm adjusts its capacity.

Proposition 3.2: The value of firm 2, the inflexible firm, is entirely determined by the value of the assets in place net of the present value of fixed costs, i.e.,

$$V^2(K_t, X_t) = \frac{R_{11}^2 X_t}{\delta} + B_1^2 X_t^{\nu_1} + B_2^2 X_t^{\nu_2} - \frac{f_1}{r} \tag{10}$$
$$= V_{AM}^2(K_t, X_t) + V_{AC}^2(K_t, X_t) + V_F^2(X_t). \tag{11}$$

where $B_1^2 \leq 0$ and $B_2^2 \geq 0$ are constants determined by the boundary conditions.

Proof: See Appendix A.

In turn, the value of the assets in place can be thought of as having three parts. One is the present value of the revenue stream assuming that there are no changes (capacity adjustments), and it is denoted by

$$V_{AM}^2(K_t, X_t) = \frac{R_{11}^2 X_t}{\delta}. \tag{12}$$

The second component is the present value of fixed costs associated with a given level of capacity,

$$V_F^2(K_t, X_t) = -\frac{f_1}{r}. \tag{13}$$

The third component is the present value of the revenue gain or loss due to the potential capacity changes elsewhere in the industry. This present value reflects the random time at which the flexible firm adjusts capacity and it has the form [5]:

$$V_{AC}^2(K_t, X_t) = B_1^2 X_t^{\nu_1} + B_2^2 X_t^{\nu_2}. \tag{14}$$

Since the value matching conditions for the inflexible firm imply that $B_1^2 \leq 0$ and $B_2^2 \geq 0$ hold, the value of the inflexible firm has a positive part associated with the put option (contraction) the flexible firm exercises when demand gets low, and a negative part related to the call option when the flexible firm expands and industry prices decrease. These two different forces will also be important when we derive dynamic betas for the flexible and inflexible firm.

[5] See Karlin and Taylor (1973) or Dixit and Pindyck (1994).

The explicit valuation formulas for the flexible and the inflexible firm allow us to derive dynamic betas for both firms following standard arguments. Beta for each firm can be calculated as the elasticity of firm value with respect to X_t, i.e.,

$$\beta^i(K_t, X_t) = \frac{\frac{\partial V^i(X)}{\partial X} X}{V^i(X)}, \; i = 1, 2. \tag{15}$$

Proposition 3.3 a) Dynamic beta for the flexible firm, firm 1, is given by

$$\beta^1(K_t, X_t) = 1 + \left\{ (\nu_1 - 1) \frac{B_1^1 X_t^{\nu_1}}{V^1(K_t, X_t)} + (\nu_2 - 1) \frac{B_2^1 X_t^{\nu_2}}{V^1(K_t, X_t)} \right\} + \frac{f_1/r}{V^1(K_t, X_t)}. \tag{16}$$

b) Dynamic beta for the inflexible firm, firm 2, is given by

$$\beta^2(K_t, X_t) = 1 + \left\{ (\nu_1 - 1) \frac{B_1^2 X_t^{\nu_1}}{V^2(K_t, X_t)} + (\nu_2 - 1) \frac{B_2^2 X_t^{\nu_2}}{V^2(K_t, X_t)} \right\} + \frac{f_1/r}{V^2(K_t, X_t)}. \tag{17}$$

Beta for the flexible firm consists of three parts. By assumption revenue beta is equal to 1. The second part is option risk and the third part is risk arising from operating leverage. Operating leverage is risk increasing. Since $B_1^1, B_2^1 \geq 0$ option risk has both a positive ($\nu_1 > 1$) and a negative component ($\nu_2 < 0$). The expansion (call) option inflates risk while the shrinkage (put) option decreases risk.

Beta for the inflexible firm also consists of three parts, revenue beta is equal to 1, operating leverage that is risk increasing and option like risk. Although the analytical structure of beta for the inflexible firm is the same as for the flexible one, the economic interpretation is very different. The flexible firm's risk depends only on firm specific decisions. In contrast, the inflexible firm, by construction, has no firm specific decisions to make. The risk dynamics are entirely due to industry effects. Furthermore, the inflexible firm risk is lower than it would be if it had no competitor. Intuitively, competitors expansion and contraction decisions dampen demand shocks. Near the expansion boundary, increases in the demand level are accompanied by a greater likelihood that the flexible firm will add capacity, increase output, and thereby mitigate the positive impact of a demand increase. Similarly, near the contraction boundary, the likelihood that the flexible firm will reduce capacity increases when demand levels fall, again offsetting demand shocks.

This simple case is illustrated in Figure 1 where, in order to highlight the importance of industry and own effects we have assumed that the inflexible firm also does not have operating leverage and we illustrate the risk of the flexible firm with and without operating leverage. X_C is the critical level of demand at which the flexible firm shrinks and X_E is critical level at which the flexible firm expands. Starting from demand level such as X_M consider a series of increases in demand. As demand increases, operating leverage decreases but option leverage increases in importance. At X_E the flexible firm expands, extinguishes its option and replaces the option with more capacity and operating leverage. As demand increases above X_E operating leverage risk decreases. Starting from demand level X_M the same path of demand increases would lead to a decrease the risk of the inflexible firm due to the fact that further positive demand shocks will induce the flexible rival to increase capacity and dampen demand shocks. Once the flexible firm has expanded, the industry is left with two firms who are not able to offset demand shocks further so the inflexible firm's risk reflects underlying demand risk which is invariant to the level of demand. On the other hand, as demand decreases from X_M both firms experience a decrease in risk in the absence of operating leverage due to the fact that further decreases in demand could induce an offsetting decrease in capacity. With operating leverage, however, decreases in demand increase risk so the net effect could be an increase in risk.

An important point to note in this diagram is the own and rival firm risk levels can move in opposite directions. As demand increases, the flexible firm's risk increases while the inflexible firm's risk decreases. If the rival is thought of as the industry, then own and industry risk can move in opposite directions as well.

In the next two sections we generalize the results found in this section by allowing two flexible firms. In section 3.2 we consider the case where both firms have an expansion option each, while in Section 3.3 we consider the case of each firm having a contraction option.

3.2 Equilibrium Exercise of Expansion Options

In this subsection we relax the assumption that there is a flexible and an inflexible firm and allow both firms to expand capacity. In the next subsection, we consider the case where both firms are able to reduce capacity through a contraction. We treat the expansion and contraction options separately as it allows us to more clearly present the strategic interactions that are involved.

Option exercise in this setting is determined by equilibrium play of both firms. In principle there are three different types of equilibria when firms have asymmetric investment costs. There is sequential exercise of options (the low cost firm exercises first and the high cost firm second), and there is preemption and simultaneous exercise of options.

Although we derive equilibrium play that includes preemption and sequential exercise, we are primarily interested in the equilibrium with sequential exercise. This implies that one of the firms must act as the leader and the other one as the follower. It turns out that in a sequential equilibrium the low cost firm is the leader and the high cost firm is the follower. We make use of standard dynamic programming techniques to derive optimal value functions for the different cases.

In an equilibrium in which asymmetric firms exercise their options sequentially, industry structure can be thought of as consisting of three distinct phases, a juvenile industry, an adolescent industry, and a mature industry. Figure 2 depicts the different industry stages.

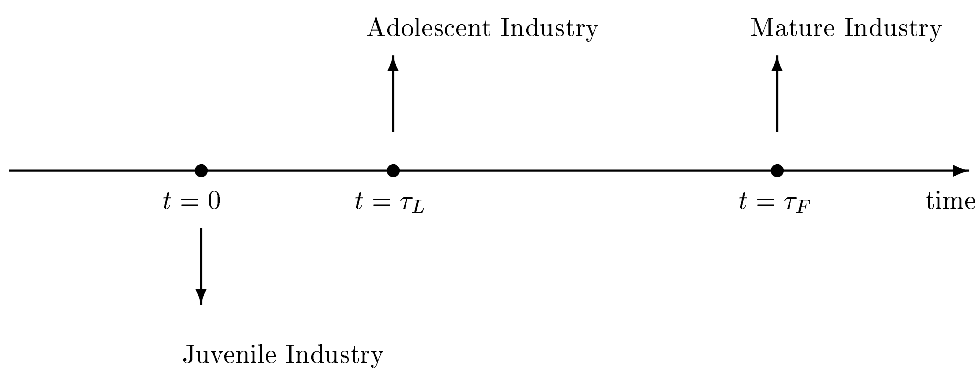

Figure 2

To achieve a specification for our general model where both firms only have a single growth option we set the capital adjustment costs to

$$\Lambda^1 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -I \\ -\infty & -\infty & 0 \end{bmatrix} \qquad \Lambda^2 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -\rho I \\ -\infty & -\infty & 0 \end{bmatrix}, \tag{18}$$

where we assume that $\rho \geq 1$ so that the costs of firm 1 are lower than the costs of firm 2, $I^1 \leq I^2$.

Based on the time line of option exercise we apply backward induction to derive the value functions for the two firms in the duopolistic industry. In a mature industry firm values equal the present value of profits minus the present value of fixed costs. In an adolescent industry the leader has already exercised his option but the follower needs to decide when optimally to exercise his option. Since both firms are operating in an imperfectly competitive output market the firm value of the leader necessarily depends on the exercise strategy of the follower. In a juvenile industry neither firm has exercised their option so that both firms have to take into account the value implications of option exercise by the leader and the follow on exercise by the follower.

To derive equilibrium behavior for our model it turns out that the value of ρ plays a crucial role. This relative investment cost difference determines which behavior corresponds to equilibrium play, i.e., sequential exercise or preemptive behavior. In deriving equilibrium play for our example we follow the approach of Pawlina and Kort (2006). They also deal with equilibrium option exercise in an asymmetric cost scenario.

Proposition 3.4 a) There exists a level of $\rho^* > 1$ such that for all $\rho \geq \rho^*$ the unique equilibrium of the investment game is that the low cost firm, firm 1, acts as the leader and the high cost firm, firm 2, acts as the follower.
b) For $1 \leq \rho < \rho^*$ any equilibrium of the investment game results in preemptive investment, in which the leader preempts the follower. In case of symmetric costs there is no pure strategy Nash equilibrium, and the only equilibrium that exists is in mixed strategies.

Proof: See Appendix B.
Proposition 3.4 allows us to to concentrate on sequential exercise of options with firm 1 being the leader and firm 2 being the follower. The leader exercises his growth option at time τ_L or equivalently at the demand trigger

X_E^1, the follower exercises at time τ_F or the demand trigger X_E^2. It holds that $\tau_L < \tau_F$ and equivalently $X_E^1 < X_E^2$. Given these two sequential exercise dates (trigger levels) the value functions of each firm consists of three components, (i) firm value in the juvenile industry ($X_t < X_E^1$), (ii) firm value in the adolescent industry ($X_E^1 \leq X_t \leq X_E^2$), and (iii) firm value in the mature industry ($X_t > X_E^2$).

The firm values for both firms are derived now. As pointed out we assume that firm 1 is the leader who invests at the option trigger X_E^1, and firm 2 is the follower who chooses capacity expansion at the trigger X_E^2.

Lemma 3.1 a) Suppose firm 1 acts as the leader and let X_E^1 be the investment trigger of the leader and X_E^2 that of the follower. The leader's value function is given by

$$V^1(K_t, X_t) = \begin{cases} \frac{R_{11}^1 X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + rI^1}{r(\nu_1 - 1)} \left(\frac{X_t}{X_E^1}\right)^{\nu_1} & \\ + \frac{X_E^2}{\delta}\left[R_{22}^1 - R_{21}^1\right]\left(\frac{X_t}{X_E^2}\right)^{\nu_1} & X_t < X_E^1, \\ \frac{R_{21}^1 X_t}{\delta} - \frac{f_2}{r} & \\ + \frac{X_E^2}{\delta}\left[R_{22}^1 - R_{21}^1\right]\left(\frac{X_t}{X_E^2}\right)^{\nu_1} & X_t \in [X_E^1, X_E^2], \\ \frac{R_{22}^1 X_t}{\delta} - \frac{f_2}{r} & X_t > X_E^2, \end{cases} \tag{19}$$

with the optimal expansion trigger X_E^1 equal to

$$X_E^1 = \frac{\delta \nu_1 (f_2 - f_1 + rI^1)}{(\nu_1 - 1) r [R_{21}^1 - R_{11}^1]} > 0. \tag{20}$$

b) Suppose firm 2 acts as the follower with the two expansion triggers X_E^1 and X_E^2. The follower's value function is given by

$$V^2(K_t, X_t) = \begin{cases} \frac{R_{11}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + rI^2}{r(\nu_1 - 1)} \left(\frac{X_t}{X_E^2}\right)^{\nu_1} & \\ + \frac{X_E^1}{\delta}\left[R_{21}^2 - R_{11}^2\right]\left(\frac{X_t}{X_E^1}\right)^{\nu_1} & X_t \leq X_E^1, \\ \frac{R_{21}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + rI^2}{r(\nu_1 - 1)} \left(\frac{X_t}{X_E^2}\right)^{\nu_1} & X_t \in [X_E^1, X_E^2], \\ \frac{R_{22}^2 X_t}{\delta} - \frac{f_2}{r} & X_t > X_E^2. \end{cases} \tag{21}$$

with the optimal expansion trigger X_E^2 equal to

$$X_E^2 = \frac{\delta \nu_1 (f_2 - f_1 + rI^2)}{(\nu_1 - 1) r [R_{22}^2 - R_{21}^2]} > 0. \tag{22}$$

Proof: See Appendix B.

It is important to note that the optimal expansion trigger of the leader (20) only depends on the leader's own characteristics and actions. There is neither a direct nor an indirect influence coming from the follower. Firm value of the leader is composed of the value of the assets in place, the value of the fixed costs, the option value, and the value correction arising from the anticipated changes resulting from the follower's option exercise. Before we study the leader's firm value in more detail we next turn to the derivation of the follower's value function.

Again we want to point out that the optimal expansion trigger of the follower is only determined by the follower's fundamentals and actions. It is independent of the leader's investment strategy.

To explore the different characteristics of the leader's and follower's value functions consider the juvenile industry. It is helpful to rewrite the trigger levels X_E^1 and X_E^2. From (20) we get

$$\frac{X_E^1}{\delta \nu_1}[R_{21}^1 - R_{11}^1] = \frac{(f_2 - f_1 + rI^1)}{(\nu_1 - 1)r} > 0$$

and from (22) we get

$$\frac{X_E^2}{\delta \nu_1}[R_{22}^2 - R_{21}^2] = \frac{(f_2 - f_1 + rI^2)}{(\nu_1 - 1)r} > 0.$$

Substituting these expressions into the value functions of the leader (19) and the follower (21) we arrive at the following. The leaders value function is given by

$$\begin{aligned} V^1(K_t, X_t) \quad = \quad & \underbrace{\frac{R_{11}^1}{\delta} X_t - \frac{f_1}{r}}_{\textbf{assets in place}} \; + \underbrace{\frac{X_E^1}{\delta \nu_1}[R_{21}^1 - R_{11}^1]\left(\frac{X_t}{X_E^1}\right)^{\nu_1}}_{\textbf{growth option}} \\ & + \underbrace{\frac{X_E^2}{\delta}[R_{22}^1 - R_{21}^1]\left(\frac{X_t}{X_E^2}\right)^{\nu_1}}_{\textbf{value adjustment}} \end{aligned}$$

and that of the follower is given by

$$V^2(K_t, X_t) = \underbrace{\frac{R^2_{11}}{\delta}X_t - \frac{f_1}{r}}_{\textbf{assets in place}} + \underbrace{\frac{X^1_E}{\delta}[R^2_{21} - R^2_{11}]\left(\frac{X_t}{X^1_E}\right)^{\nu_1}}_{\textbf{value adjustment}}$$
$$+\underbrace{\frac{X^2_E}{\delta\nu_1}[R^2_{22} - R^2_{21}]\left(\frac{X_t}{X^2_E}\right)^{\nu_1}}_{\textbf{growth option}}$$

Each value function in a juvenile industry consists of four parts that can be given the following interpretation. In both cases (leader and follower) the first part measures the value of the assets in place assuming that the firms use their initial levels of capacity. The second part measures the value of the fixed costs. The third term in the leader's valuation is the option value that is only related to the leader's own action (option exercise). The fourth term in the leader's value function is the value adjustment of the assets in place after the follower's option exercise. Off course this value adjustment is negative, because an increase in capacity by the rival causes the market price to drop. Since the leader's assets in place are at the capacity level κ_2 this value adjustment can be viewed as an adjustment to the value of the assets in place induced by an action of the rival firm. Denoting this value adjustment by $V^1_{AC}(K_t, X_t)$ we can rewrite the leaders value function as

$$V^1(K_t, X_t) = V^1_{AM}(K_t, X_t) + V^1_{AC}(K_t, X_t) + V^1_{OM}(K_t, X_t) + V^1_F(K_t, X_t). \quad (23)$$

Let us next turn to the value function of the follower. As pointed out above the first two value components of the follower's value function are the value of the assets in place and the value of the fixed costs. The third part in the follower's value is now driven by the leader's action. It is the value adjustment necessary to account for the change in the follower's revenues when the leader exercises the option and the market price drops. As with the leader's value function this is negative. The fourth part of the follower's value function is the option value that accounts for the increase in capacity and takes into account the corresponding revenue change. Taking all the four value drivers together we get

$$V^2(K_t, X_t) = V^2_{AM}(K_t, X_t) + V^2_{AC}(K_t, X_t) + V^2_{OM}(K_t, X_t) + V^2_F(K_t, X_t). \quad (24)$$

We are now in a position to summarize the valuation of both the leader and the follower and state the following result.

Proposition 3.4 The firm values of the leader, firm 1, and the follower, firm 2, are given by

$$V^i(K_t, X_t) = V^i_{AM}(K_t, X_t) + V^i_{AC}(K_t, X_t) + V^i_{OM}(X_t) + V^i_F(K_t, X_t), i = 1, 2. \tag{25}$$

This result is an immediate consequence of our preceding discussion. The value functions for both firms can now be used to derive the risk implications in an industry in which each firm has a single growth option. Our risk analysis can either be based on the individual firm or on the industry level. Industry beta is defined as the beta of a portfolio that consists of both firms in the industry.

Proposition 3.5 Consider a growing industry with sequential exercise of options in which each firm has a single expansion option. Systematic firm risks for both the follower ($i = 2$) and the leader ($i = 1$) over different industry stages ($k = 1, 2$) are given by

$$\beta^i(K_t, X_t) = 1 + \frac{V^i_{OM}(K_t, X_t) + V^i_{AC}(K_t, X_t)}{V^i(K_t, X_t)}(\nu_1 - 1) + \frac{f_k/r}{V^i(K_t, X_t)}, \; k = 1, 2 \tag{26}$$

where $V^i_{AC}(K_t, X_t) < 0$ holds. Hence, strategic competition is risk reducing.

The results in Proposition 3.5 are surprising and have the following implications. As stated, competition is risk reducing. This is counter intuitive but is the consequence of $V^i_{AC}(K_t, X_t) < 0$ which reflects a hedging role for competition. The intuition supporting this hedging role can be seen by first considering an industry where firms can neither expand nor contract. Demand shocks translate directly into price shocks. Now allow either or both firms to expand in response to a demand shock. In this case, when the demand shock causes the expansion trigger to be met, the price increase that would come from the increase in demand is offset by the supply increase that comes from the increase in supply. The supply response, therefore, provides a hedge against some of the demand uncertainty. This hedge is larger the closer the follower comes to exercising his growth option.

Figure 3 gives a graphical presentation of the hedging argument. Before the follower exercises his growth option, industry output is given by the level

Q_1. Since both firms have to produce at full capacity levels price fluctuates along the supply curve Q_1. Let's suppose that demand increases by an efficient amount so that the follower finds it optimal to exercise his growth option. Option exercise results in an increase in industry output to the level Q_2. The increase in industry supply causes prices to increase less than to the level indicated by the old supply curve P^*. The new price level is P_2 instead of P^*. This dampening corresponds to the hedging effect.

Systematic firm risk in a growing oligopolistic industry is driven by the growth option (i.e. the size of the firm), by operating leverage (i.e. firm's book to market), and an industry effect (the strategic effect). While the growth option and operating leverage are risk increasing, the strategic or industry effect is risk reducing. Hence, we find that firm's own and industry characteristics can have opposite risk implications.

Risks dynamics are sketched in Figure 4. For ease of exposition let us look at the leader. Prior to the exercise of the growth option risk of the leader is running up until the exercise trigger is reached. Immediately after the exercise and prior to the exercise of the follower's expansion option, risk of the leader drops below market risk which is normalized by 1 and deceases until the follower exercises his option. After both firms have exercised their options risk is only driven by operating leverage.

3.3 Equilibrium Exercise of Contraction Options

The risk analysis of a growing industry in which firms exercise growth options has revealed two important results. We found that industry effects arising from the strategic interactions of rival firms are risk reducing and that firm's own and industry characteristics have opposite risk implications. Moreover, we have found that equilibrium play crucially depends on the investment cost differences of the two rival firms. If the cost difference is large enough it follows that there exists a unique Nash equilibrium with sequential exercise of options. If the costs difference is not too high, there are preemptive equilibria and in case of symmetric costs ($\rho = 1$) there only exists a Nash equilibrium in mixed strategies (see Boyer et al. (2007)).

In this section we assume that each firm has a single contraction option. Firms are operating with a given initial capacity κ_1 and that each has a single option to reduce capacity to a level given by $\kappa_0 < \kappa_1$. Contraction to a smaller firm size and capacity will be optimal for the firms if demand turns out to be low so that existing capacity levels cannot be sustained. Allowing for contraction options can be seen as a substitute for operating flexibility. Operating flexibility implies that firms can adjust their output levels downwards and omit some idle capacities when industry demand is low.

To achieve a specification where both firms only have a single contraction option we set the capital adjustment costs to

$$\Lambda^1 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ S & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix} \qquad \Lambda^2 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ \rho S & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix}, \qquad (27)$$

where we assume that $0 < \rho \leq 1$, so that firm 1 has the high and firm 2 the low salvage value, $S^2 < S^1$. This implies that firm 1 has an incentive to contract earlier. Our interest again lies in equilibrium play of the two rivals and it turns out that, contrary to the case of expansion option, sequential exercise is the unique equilibrium as long as there is a difference in the salvage values, i.e., $\rho < 1$.

Proposition 3.6 For every $0 < \rho < 1$ there exists a unique equilibrium of the contraction game in which the high salvage value firm, firm 1, acts as the leader and the low salvage value firm, firm 2, acts as the follower.

Proof: See Appendix C.

The last result is very interesting and establishes an asymmetric role of expansion and contraction. The result is driven by the fact that the strategic effect of a contraction is not negative, because the follower profits from a capacity contraction of the leader. That the leader has an incentive to exercise first is the consequence of the higher salvage value. Hence, there is no preemptive role for the players in the contraction game. This result was also found by Murto (2004). Hence, we discuss the sequential contraction case only.

As in the case of growth options we start out with an industry in which both firms produce at capacity levels equal to κ_1. This industry stage is followed by an industry in which the leader shrinks to the capacity level κ_0 while the follower still keeps producing at κ_1. The last stage is the industry in which both firms have reduced their capacity to κ_0.

In the sequential industry the leader, firm 1, contracts first at the demand trigger X_C^1, and the follower, firm 2, second at the trigger X_C^2. It holds $X_C^1 > X_C^2$.

Lemma 3.2 Suppose firm 1 is the leader and let X_C^1 be the contraction trigger of the leader and X_C^2 that of the follower. The leader's value function is given by

$$V^1(K_t, X_t) = \begin{cases} \frac{R_{11}^1 X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + rS^1}{r(1-\nu_2)} \left(\frac{X_t}{X_C^1}\right)^{\nu_2} & \\ +\frac{X_C^2}{\delta}\left[R_{00}^1 - R_{01}^1\right]\left(\frac{X_t}{X_C^2}\right)^{\nu_2} & X_t > X_C^1, \\ \frac{R_{01}^1 X_t}{\delta} - \frac{f_0}{r} & \\ +\frac{X_C^2}{\delta}\left[R_{00}^1 - R_{01}^1\right]\left(\frac{X_t}{X_C^2}\right)^{\nu_2} & X_t \in [X_C^2, X_C^1], \\ \frac{R_{00}^1 X_t}{\delta} - \frac{f_0}{r} & X_t < X_C^2, \end{cases} \tag{28}$$

with a contraction trigger X_C^1 equal to

$$X_C^1 = \frac{\delta\nu_2(f_1 - f_0 + rS^1)}{(1-\nu_2)r[R_{01}^1 - R_{11}^1]} > 0. \tag{29}$$

Proof: See Appendix C.

The sign of the optimal contraction trigger (29) of the leader is determined by the properties of the revenue factors. The assumption on the price elasticity of the inverse demand function implies that

$$R_{01}^1 < R_{11}^1$$

holds. Hence X_C^1 is strictly positive. Rewriting the contraction trigger of the leader as

$$\frac{X_C^1}{\delta\nu_2}[R_{01}^1 - R_{11}^1] = \frac{(f_1 - f_0 + rS^1)}{(1-\nu_2)r} > 0 \tag{30}$$

and substituting this into the value function (28) results in

$$\begin{aligned} V^1(K_t, X_t) \quad = \quad & \underbrace{\frac{R_{11}^1}{\delta}X_t - \frac{f_1}{r}}_{\textbf{assets in place}} + \underbrace{\frac{X_C^1}{\delta\nu_2}[R_{01}^1 - R_{11}^1]\left(\frac{X_t}{X_C^1}\right)^{\nu_2}}_{\textbf{contraction option}} \\ & + \underbrace{\frac{X_C^2}{\delta}\left[R_{00}^1 - R_{01}^1\right]\left(\frac{X_t}{X_C^2}\right)^{\nu_2}}_{\textbf{value adjustment}}. \end{aligned}$$

The value function has four elements. The first one measures the value of the assets in place starting with the initial capacity level. The second one measures the value of the fixed costs. Both values are the outcome of the firm's own actions. The third element measures the value of the contraction option. This corresponds to the positive value of a put option as indicated by ν_2. The last term measures the value adjustment arising from the contraction of the follower after the leader already has contracted. This term has a positive value because any reduction in the capacity of the rival firm causes the market price to increase which benefits the leader who already contracted. This last term, however, is not the outcome of the firm's own action but has to be attributed to the actions of the rival firms. By combining these elements we obtain

$$V^1(K_t, X_t) = V^1_{AM}(K_t, X_t) + V^1_{AC}(K_t, X_t) + V^1_{OM}(K_t, X_t) + V^1_F(K_t, X_t). \quad (31)$$

The industry effect derived for the leader, also applies to the follower. Prior to the exercise of the put option of the leader, the follower's value also depends on the strategic effect.

Lemma 3.3 Suppose firm 2 is the follower and let X^1_C be the contraction trigger of the leader and X^2_C that of the follower. The follower's value function is given by

$$V^2(K_t, X_t) = \begin{cases} \frac{R^2_{11} X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + rS^2}{r(1-\nu_2)} \left(\frac{X_t}{X^2_C}\right)^{\nu_2} & \\ \frac{X^1_C}{\delta} \left[R^2_{01} - R^2_{11}\right] \left(\frac{X_t}{X^1_C}\right)^{\nu_2} & X_t \geq X^1_C, \\ \frac{R^2_{01} X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + rS^2}{r(1-\nu_2)} \left(\frac{X_t}{X^2_C}\right)^{\nu_2} & X_t \in [X^2_C, X^1_C], \\ \frac{R^2_{00} X_t}{\delta} - \frac{f_0}{r} & X_t < X^2_C. \end{cases} \quad (32)$$

with a contraction trigger X^2_C equal to

$$X^2_C = -\frac{\delta \nu_2 (f_1 - f_0 + rS^2)}{(1-\nu_2) r [R^2_{00} - R^2_{01}]} > 0. \quad (33)$$

Proof: See Appendix C.

Again is should be pointed out that $R^2_{00} < R^2_{01}$ which ensures that also the contraction trigger of the follower is strictly positive. Summing up again, the firm value of the follower is given by the value of the assets in place, the value of the fixed costs, the contraction option and the value adjustment related to the option exercise of the leader. To see this value decomposition let us rewrite again the trigger level of the follower as

$$\frac{X_C^2}{\delta\nu_2}[R_{00}^2 - R_{01}^2] = \frac{(f_1 - f_0 + rS^2)}{(1-\nu_2)r} > 0.$$

Hence, the value function for the follower becomes

$$\begin{aligned} V^2(K_t, X_t) \quad = \quad & \underbrace{\frac{R_{11}^2}{\delta}X_t - \frac{f_1}{r}}_{\textbf{assets in place}} + \underbrace{\frac{X_C^2}{\delta\nu_2}[R_{00}^2 - R_{01}^2]\left(\frac{X_t}{X_C^2}\right)^{\nu_2}}_{\textbf{contraction option}} \\ & + \underbrace{\frac{X_C^1}{\delta}\left[R_{01}^2 - R_{11}^2\right]\left(\frac{X_t}{X_C^1}\right)^{\nu_2}}_{\textbf{value adjustment}}. \end{aligned}$$

This value function exhibits all the four different value elements discussed above. Hence, we are in a position to state our next result.

Proposition 3.7 The firm values of the leader, firm 1, and the follower, firm 2, are given by

$$V^i(K_t, X_t) = V_{AM}^i(K_t, X_t) + V_{AC}^i(K_t, X_t) + V_{OM}^i(K_t, X_t) + V_F^i(K_t, X_t). \quad (34)$$

Compared to the results derived for the case of expansion options the strategic effect in case of downsizing is positive $V_{AC}^i(K_t, X_t) > 0$ and hence value increasing. But as we now demonstrate, it is also risk reducing.

Proposition 3.8 Systematic firm risks for both the follower ($i = 2$) and the leader ($i = 1$) over the different industry stages are given by

$$\beta^i(t) = 1 + \frac{V_{OM}^i(K_t, X_t) + V_{AC}^i(K_t, X_t)}{V^i(K_t, X_t)}(\nu_2 - 1) + \frac{f_k/r}{V^i(K_t, X_t)}, \ k = 1, 0. \quad (35)$$

where $\nu_2 < 0$ and $V_{OM}^i(K_t, X_t) + V_{AC}^i(K_t, X_t) > 0$ hold. Hence, again competition is risk reducing. The last result demonstrates that the strategic effect is again risk reducing so that competition in our model is risk reducing, independent of whether we are in a growing or a shrinking industry.

4 Empirical Analysis of Rival Effects on Returns

In this section, we empirically examine the intuition developed in the previous sections where we show that both own-firm and rival-firm characteristics should drive risk and return. In a bi-variate analysis, we relate return means, variances, and market betas to firm and industry book-to-market. A multivariate analysis using Fama-MacBeth regressions illustrates how firm and industry beta, size, and book-to-market impact on expected returns.

4.1 Own and Industry Book-to-market Effects

Following Fama and French (1992), data on market prices from CRSP and annual book-equity from Compustat are used to assign market values and book-to-market ratios (B/M) to every available firm. These firms are additionally assigned to one of 49 industries using the classification scheme of Fama and French (1997). We interpret own firm B/M as a proxy for the state of own-firm demand and capital levels and, similarly, the market-value weighted-average of firm B/M within the same industry as a proxy for demand and capital states of rivals.

Using annual sorts based on own and industry B/M, 25 value-weighted portfolios are formed. Table 1 reports the average of monthly returns for these portfolios during the time period July 1963 - December 2005, using both independent sorts (Panel A), as well as dependent sorts (Panels B and C). The table shows the well-known value effect, whereby own-firm B/M and expected returns are positively related. For example, Panel A shows that the high B/M value firms produce expected returns as high as 2.11% per month, while the low B/M growth firms have much lower expected returns, as low as 0.68% per month.

Consistent with the hypothesis that rival actions can affect firm risk, Table 1 shows that industry B/M has a significant impact on expected returns. The industry effect is opposite to the own-firm effect, since industry B/M and expected returns are negatively related. Thus, holding own-firm characteristics constant, increases in rival B/M cause decreases in expected returns.

Table 2 shows how firm risk, as measured by realized volatility using daily returns within a year, are related to own and industry B/M. Own-firm volatility is relatively high when B/M is either high or low, consistent with the U-shaped pattern relating own B/M to volatility in Carlson, Fisher, and Giammarino (2004). Industry B/M has a robustly monotonic impact on firm risk, with high levels of B/M predicting low volatility. Table 3 further supports the risk-reducing impact of industry B/M. Conditioning first on own-firm B/M, an increase in industry B/M predicts a decrease in beta.

4.2 Fama-MacBeth Regressions

To study the separate impacts of firm and rival characteristics on expected stock returns, we analyze monthly Fama-MacBeth regressions of individual firm returns on appropriately lagged own-firm and industry beta, size, and B/M. Betas for each firm are calculated annually using contemporaneous, leading and lagged daily returns.

Table 4 presents three sets of results: Regressions (1)-(4) use own-firm characteristics, (5)-(8) use industry characteristics, and (9)-(13) use both. Regressions (1)-(4) are consistent with results from Fama and French (1992), highlighting the ineffectiveness of beta when combined with size and B/M. Regressions (6)-(8) show that industry characteristics alone are ineffective predictors of the cross-sectional dispersion of firm expected returns.

Our main interest is in the regressions with both own-firm and industry characteristics. In general, the signs on own-firm and industry characteristics are opposite. For example, regression (11) confirms the results in the previous subsection by identifying both a significant positive impact of own B/M and a significant negative impact of industry B/M on expected returns. Industry size also exhibits this pattern when all six explanatory variables are included.

Industry effects are further decomposed in regression (13), where a firm's own share (OS) is interacted with own-firm characteristics and industry characteristics. In this regression, the variable OS is given by the fraction of industry sales in a given year that are generated by an individual firm, and industry characteristics are weighted to exclude the firm's own characteristics (the "VWX" characteristics). The industry characteristics as utilized in the prior regressions are dropped from the regression since they are approximately equal to the own- and industry-share weighted averages of own and VWX values.

To interpret the coefficients in this regression, it is helpful to consider the separate impacts of operating leverage and expansion or contraction decisions on expected returns. When own firm B/M is high, operating leverage and, consequently, expected returns are high. This effect holds regardless of how important the firm is in terms of product market impact. As our model with contraction options shows, the product market impact of exit is to dampen risky demand shocks and reduce risk. This impact is relatively more important when a firm is a significant source of supply in its industry. This effect should, in principle, offset the leverage component in the B/M proxy and is in addition to the rival B/M effects. The coefficients in the Fama-MacBeth regressions are consistent with this hypothesis. Own B/M has a positive coefficient, whereas the coefficients on the OS and rival B/M ratios are negative.

Overall, our empirical results provide robust evidence that rival characteristics have an important impact on firm returns. This analysis provides new direct evidence on the risk-reducing impact of industry B/M on firm returns.

5 Conclusion

In this paper we consider a duopolistic industry with firms producing a homogenous product at given capacity levels. Demand in the industry is stochastic and governed by an industry shock that follows a geometric Brownian motion. Firms produce with given capacity levels that are fixed (i.e. there is no operating flexibility) but can increase (decrease) their output with the exercise of a growth (contraction) option. Although there are no variable production costs, firms operate with fixed costs that change with the level of capacity. Growth option exercise causes the firms to incur investment costs that include adjustment costs and the price of the investment, while in case of the contraction option firms incur a salvage value. We assume that one of the firms is a high cost (high salvage value) and one a low cost (low salvage value) firm. In terms of option exercise we only consider the case of sequential exercise. The low cost (high salvage value) firm acts as the leader and exercises first and the high cost (low salvage value) firm acts as the follower and exercises second. We discuss how this exercise behavior corresponds to a subgame perfect Nash equilibrium.

Given this industry structure we derive firm values and risk dynamics for individual firms and their rivals. We find interesting novel results. First, we identify a strategic effect that reduce firm risk. Hence, relative to a monopolistic situation, we argue that more competition results in lower firm risk. This reduced risk is the consequence of a hedging effect. In case both firms operate with fixed capacity levels any profit uncertainty arises from the industry demand shock. Demand shocks directly translate into changes in the firm's cash flows. If, however, the leader who already exercised his growth option and faces fixed capacity forever, faces an change in the capacity of the follower upon the follower's option exercise, demand shocks are hedged by an output increase or decrease, depending whether we are in a growing or shrinking industry. This hedge is larger the closer the follower comes to exercising his option. As a consequence, the leaders risk is reduced and is below the market risk normalized by 1. This is a counter-intuitive result and does not confirm the empirical findings of Hou and Robinson (2005). The result, however, allows for a better understanding of the driving forces behind a firm's risks. As pointed out, many existing studies identify operating leverage and irreversibility as the two channels that drive the risk dynamics in a market. In this paper we add an additional factor, that we call industry factor. It turns out, that the industry factor behaves differently, depending on the whether the industry grows or shrinks. In case of expansion options, firm own and industry characteristics have opposite risk implications. In case of a contraction option, firm own and industry characteristics have the same risk implications.

There are many open questions for further research. It is important to allow for operating flexibility and derive the risk dynamics in this case. When firms have operating flexibility they need not produce with a given capacity level but can choose to have idle capacities if demand is low. It is clear that operating flexibility substantially changes the risk dynamics. Additionally, it seems interesting to look at risk dynamics in an equilibrium with simultaneous exercise. This simultaneous exercise can exist even if firms are asymmetric. Finally, it is a challenge to allow for a more dynamic capacity expansion in which each firm faces several growth options to expand.

Appendix

A Proofs

Proof of Proposition 3.1: To derive the value of the flexible firm we assume that there exist two traded assets that can be used to hedge industry demand uncertainty. Let B_t denote the price of a riskless bond with dynamics $dB_t = rB_t dt$ where $r > 0$ is the constant riskless rate of interest, and let S_t be the price of a risky asset. The price dynamics of the risky asset is given by

$$dS_t = \mu S_t dt + \sigma S_t dW_t. \tag{36}$$

The risky asset S_t and the industry demand shock X_t are perfectly correlated. Hence, we can use the securities B_t and S_t, to construct a portfolio of the bond and the asset S_t that perfectly replicates the industry shocks X_t and derive its risk neutral measure. Demand dynamics under risk neutral measure are given by

$$dX_t = (r - \delta)X_t dt + \sigma X_t d\hat{W}_t, \tag{37}$$

where $\delta \equiv \mu - g > 0$. All the valuations in this paper are based on the risk neutral measure (37).

Under the risk neutral measure (37) the value of the flexible firm needs to satisfy the valuation equation

$$\frac{1}{2}\sigma^2 X^2 V^1_{XX} + (r - \delta)XV^1_X - rV^1 + XR^1_{11} - f_1 = 0. \tag{38}$$

with the boundary conditions

$$V^1(X_E) = \frac{R^1_{21}X_E}{\delta} - I - \frac{f_2}{r} \tag{39}$$

$$V^1(X_C) = \frac{R^1_{01}X_C}{\delta} + S - \frac{f_0}{r} \tag{40}$$

$$V^1_X(X_E) = \frac{R^1_{21}}{\delta} \tag{41}$$

$$V^1_X(X_C) = \frac{R^1_{01}}{\delta}. \tag{42}$$

The first two equations are the value matching conditions and specify that the option value at the critical boundaries are exactly equal to the present value of the incremental revenues net of adjustment costs. The last two equations are the smooth pasting conditions which are necessary for value maximization. This system of equations has no convenient analytical solution for X_E and X_C due to its nonlinearity.

Using standard techniques the solution to equation (38) is given by

$$V^1(K_t, X_t) = \frac{R_{11}^1 X_t}{\delta} - \frac{f_1}{r} + B_1^1 X_t^{\nu_1} + B_2^1 X_t^{\nu_2},$$

where B_1^1 and B_2^1 solve

$$\begin{aligned} (1-\nu_1)B_1^1 X_E^{\nu_1} + (1-\nu_2)B_2^1 X_E^{\nu_2} &= -I - \frac{f_2 - f_1}{r}, \\ (1-\nu_1)B_1^1 X_C^{\nu_1} + (1-\nu_2)B_2^1 X_C^{\nu_2} &= S - \frac{f_0 - f_1}{r}. \end{aligned}$$

For given positive values of X_E and X_C these solution satisfies $B_1^1, B_2^1 > 0$. The values X_E and X_C can only be derived numerically. But it holds that $X_E > X_C$. **qed**

Proof of Proposition 3.2: Using the risk neutral dynamics (37) the value of the inflexible firm has to satisfy the valuation equation

$$\frac{1}{2}\sigma^2 X^2 V_{XX}^2 + (r-\delta)XV_X^2 - rV^2 + XR_{11}^2 - f_1 = 0. \tag{43}$$

with the boundary conditions

$$V^2(X_E) = \frac{R_{21}^2 X_E}{\delta} - \frac{f_1}{r} \tag{44}$$

$$V^2(X_C) = \frac{R_{01}^2 X_C}{\delta} - \frac{f_1}{r}, \tag{45}$$

for given trigger levels X_E and X_C. The boundary conditions are two value matching conditions. Solving the valuation equation using the value matching conditions results in the value of the flexible firm given by

$$V^2(K_t, X_t) = \frac{R_{11}^2 X_t}{\delta} - \frac{f_1}{r} + B_1^2 X_t^{\nu_1} + B_2^2 X_t^{\nu_2},$$

where B_1^2 and B_2^2 are the solutions to the equations

$$\begin{aligned} B_1^2 X_E^{\nu_1} + B_2^2 X_E^{\nu_2} &= \frac{R_{21}^2 - R_{11}^2}{\delta} X_E - \frac{f_2 - f_1}{r} \\ B_1^2 X_C^{\nu_1} + B_2^2 X_C^{\nu_2} &= \frac{R_{01}^2 - R_{11}^2}{\delta} X_C - \frac{f_0 - f_1}{r}. \end{aligned}$$

For this equation system it is easy to show that $B_2^1 < 0$ and $B_2^2 > 0$. **qed**

Appendix B

Proof of Proposition 3.4

a) This proof follows closely that of Pawlina and Kort (2006). The sequential exercise of growth options only occurs if firm 2, the follower, does not have an incentive to be the leader. Let $V_F^2(K_t, X_t)$ be the value function of firm 2, when it acts as the follower, and let $V_L^2(K_t, X_t)$ be the value function when firm 2 acts as the leader. In the adolescent industry, when the leader already exercised his option these the value function of firm 2 when it acts as the follower becomes

$$V_F^2(K_t, X_t) = \begin{cases} \frac{R_{21}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + r\rho I}{r(\nu_1 - 1)} \left(\frac{X_t}{X_E^{2,F}} \right)^{\nu_1} & X_t \leq X_E^{2,F} \\ \frac{R_{22}^2 X_t}{\delta} - \frac{f_2}{r} - \rho I & X_t > X_E^{2,F}, \end{cases} \tag{46}$$

where $X_E^{2,F}$ is the investment trigger when firm 2 acts as the follower. Now assume, instead, that firm 2 acts as the leader. The value function of firm 2 at the time when it invests as the leader becomes

$$V_L^2(K_t, X_t) = \begin{cases} \frac{R_{12}^2 X_t}{\delta} - \frac{f_2}{r} - \rho I + \frac{X_E^{1,F} [R_{22}^2 - R_{12}^2]}{\delta} \left(\frac{X_t}{X_E^{2,F}} \right)^{\nu_1} & X_t \leq X_E^{1,F} \\ \frac{R_{22}^2 X_t}{\delta} - \frac{f_2}{r} - \rho I & X_t > X_E^{1,F}, \end{cases} \tag{47}$$

where $X_E^{1,F}$ is the investment trigger of firm 1 when firm 1 acts as the follower. Firm 2 does not have an incentive to be the leader iff

$$G(X_t, \rho) \equiv V_L^2(X_t, \rho) - V_F^2(X_t, \rho) \leq 0$$

holds.[6] The value functions (46) and (47) satisfy the properties:

$$\begin{aligned} V_F^2(K_t, X_t) & \text{ is strictly convex in } X_t \text{ for all } X_t \leq X_E^{2,F} \\ V_L^2(K_t, X_t) & \text{ is strictly concave in } X_t \text{ for all } X_t \leq X_E^{1,F}. \end{aligned}$$

Moreover, the trigger level $X_E^{2,F}$ is given by

$$X_E^{2,F} = \frac{\nu_1 \delta (f_2 - f_1 + r\rho I}{(\nu_1 - 1) r [R_{22}^2 - R_{21}^2]} \tag{48}$$

[6] Since the value functions depend on the relative cost difference ρ we explicitly use it as an argument.

and that of $X_E^{1,F}$ is given by

$$X_E^{1,F} = \frac{\nu_1 \delta(f_2 - f_1 + rI}{(\nu_1 - 1)r[R_{22}^1 - R_{12}^1]}. \tag{49}$$

Given our assumptions on the revenue function it follows that trigger (48) is greater or equal to (49) for $\rho > 1$.

As mentioned firm 2 does not have an incentive to become the leader iff $G(X_t, \rho) \leq 0$ for all $X_t \leq X_E^{2,F}$. Given the properties of $V_L^2(X_t, \rho)$ and $V_F^2(X_t, \rho)$ this holds true iff we find (X^*, ρ^*) such that

$$G(X^*, \rho^*) = 0, \tag{50}$$

$$\frac{\partial G(X^*, \rho^*)}{\partial X} = 0. \tag{51}$$

It is straight forward to show that (50) and (51) are satisfied iff

$$X^* = \frac{\nu_1}{\nu_1 - 1} \frac{\delta \rho^* I}{[R_{12}^2 - R_{21}^2]}. \tag{52}$$

(52) together with (50) can be used to derive ρ^*. At the point (X^*, ρ^*) the value functions of firm 2 acting as the follower is tangent to the value function of firm 2 acting as the leader. Hence, for all $\rho \geq \rho^*$ the follower does not have an incentive to become the leader. Therefore for this set of parameter restrictions the equilibrium outcome is that firm 1 acts as the leader and firm 2 acts as the follower. This is the unique pure strategy Nash equilibrium outcome.

b) For $\rho < \rho^*$ firm 2 has an incentive to become the leader. This incentive exists for all values of X_t in the interval $[X_E^{2,P}, X_E^{1,F}]$, where $X_E^{2,P}$ is defined by

$$V_F^2(X_E^{2,P}, \rho) = V_L^2(X_E^{2,P}, \rho).$$

We are now in the scenario of a preemptive equilibrium. In this equilibrium firm 1 will either invest at the level $X_E^{2,P}$ or at its leader trigger level X_E^1 if $X_E^1 < X_E^{2,P}$. Further details on the preemptive equilibrium and the non-existence of a pure strategy equilibrium in case of $\rho = 1$ can be found in Boyer et al. (2007). **qed**

Proof of Lemma 3.1: The value function for the leader ($i = 1$) needs to satisfy the Bellman equation

$$\frac{1}{2}\sigma^2 X^2 V_{XX}^1 + (r - \delta) X V_X^1 - rV^1 + XR_{11}^1 - f_1 = 0 \tag{53}$$

together with the boundary conditions

$$\begin{aligned} V^1(0) &= -\frac{f_0^i}{r}, \\ V^1(X_E^1) &= \frac{X_E^1 R_{21}^1}{\delta} - \frac{f_2}{r} - I + A_3^1 (X_E^1)^{\nu_1}, \\ V_X^1(X_E^1) &= \frac{R_{21}^1}{\delta} + \nu_1 A_3^1 (X_E^1)^{\nu_1 - 1}, \\ V^i(X_E^2) &= \frac{X_E^2 R_{22}^1}{\delta} - \frac{f_2}{r}. \end{aligned}$$

A general solution to the Bellman equation is given by

$$V^1(X) = A_0^1 + A_1^1 X + A_2^1 X^{\nu_1} \tag{54}$$

where $A_k^1, k = 0, 1, 2$ are constants that are determined together with the boundary conditions. The constant A_3^1 from above expresses the change of the value function for the leader after the capacity expansion of the follower has taken place. It is determined by the boundary condition

$$V^1(X_E^2) = \frac{X_E^2 R_{22}^1}{\delta} - \frac{f_2}{r}. \tag{55}$$

Solving the Bellman equation together with the boundary conditions results in

$$\begin{aligned} A_0^1 &= -\frac{f_1}{r} \\ A_1^1 &= \frac{R_{11}^1}{\delta} \\ A_2^1 &= \frac{f_2 - f_1 + rI}{r(\nu_1 - 1)} (X_E^1)^{-\nu_1} + A_3^1 \\ A_3^1 &= \frac{X_E^2 [R_{22}^1 - R_{21}^1]}{\delta} (X_E^2)^{-\nu_1} \end{aligned}$$

which results in the value function for the leader given by (19).

We proceed as above when deriving the value function for the follower

($i=2$) and note that the boundary conditions now become

$$\begin{aligned} V^2(0) &= -\frac{f_1}{r}, \\ V^2(X_E^2) &= \frac{X_E^2 R_{22}^2}{\delta} - \frac{f_2}{r} - \rho I + A_3^2 (X_E^2)^{\nu_1}, \\ V_X^2(X_E^2) &= \frac{R_{22}^2}{\delta} + \nu_1 A_3^2 (X_E^2)^{\nu_1 - 1}, \\ V^2(X_E^1) &= \frac{X_E^1 R_{21}^2}{\delta} - \frac{f_1}{r}. \end{aligned}$$

The change of the boundary conditions relative to the proof of the leader is the consequence of the follower's response to the leader's exercise of the option at the trigger level X_E^1. The constant A_3^2 accounts for this change. At the trigger level X_E^1 when the leader exercises his option the follower's value function needs to satisfy

$$V^2(X_E^1) = \frac{X_E^1 R_{21}^2}{\delta} - \frac{f_1}{r} \tag{56}$$

which implies a follower's value function equal to (21). **qed**

Appendix C

Proof of Proposition 3.6: We follow closely the argument used in the proof of Proposition 3.4. . We assume that now $\rho < 1$ so that the salvage value of firm 2 is strictly smaller than that of firm 1. This suggests that firm 2 is the follower in the contraction game, it is the firm that contracts later. The value function of firm 2 when it acts as the follower is given by

$$V_F^2(K_t, X_t) = \begin{cases} \frac{R_{01}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + r\rho S}{r(1-\nu_2)} \left(\frac{X_t}{X_C^{2,F}} \right)^{\nu_2} & X_t \geq X_C^{2,F} \\ \frac{R_{00}^2 X_t}{\delta} - \frac{f_0}{r} + \rho S & X_t \leq X_C^{2,F}, \end{cases} \tag{57}$$

where $X_C^{2,F}$ is the contraction trigger when firm 2 acts as the follower. Now assume, instead, that firm 2 acts as the leader. The value function of firm 2 at the time when it contracts as the leader becomes

$$V_L^2(K_t, X_t) = \begin{cases} \frac{R_{10}^2 X_t}{\delta} - \frac{f_0}{r} + \rho S + \frac{X_C^{1,F}[R_{00}^2 - R_{10}^2]}{\delta} \left(\frac{X_t}{X_C^{2,F}} \right)^{\nu_2} & X_t \geq X_C^{1,F} \\ \frac{R_{00}^2 X_t}{\delta} - \frac{f_0}{r} + \rho S & X_t \leq X_C^{1,F}, \end{cases} \tag{58}$$

where $X_C^{1,F}$ is the trigger when firm 1 contracts as the follower and firm 2 acts as the leader. The two contraction triggers are given by

$$X_C^{2,F} = \frac{\nu_2 \delta(f_1 - f_0 + r\rho S}{(1-\nu_2) r [R_{00}^2 - R_{01}^2]} \tag{59}$$

and

$$X_C^{1,F} = \frac{\nu_2 \delta(f_1 - f_0 + rS}{(1-\nu_2) r [R_{00}^1 - R_{10}^1]}. \tag{60}$$

Given our assumptions on the revenue function it follows that trigger (60) is strictly greater than trigger (59) for $0 < \rho < 1$. From this property and the value functions (57) and (58) it can be shown that

$$G(X_t, \rho) \equiv V_L^2(X_t, \rho) - V_F^2(X_t, \rho) \leq 0$$

holds for all X_t. Hence, firm 2 never has an incentive to become the leader. Therefore sequential exercise of contraction options is the unique pure strategy Nash equilibrium. **qed**

Proof of Lemma 3.2: The proof of this Lemma follows exactly that of Lemma 3.1 with the only change that because of the contraction the call option has to be changed to a put option with the corresponding terminal conditions.

Proof of Lemma 3.3: The proof of this Lemma follows exactly that of Lemma 3.1 with the change that we are now dealing with a contraction option that corresponds to a put rather than a call option.

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