Credit Risk and Optimal Capital Structure under Incomplete Accounting Information and Liquidity Risk

Wolfgang Bühler and Tim Thabe
University of Mannheim

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Abstract

In a structural model for credit risk we endogenize inability to pay as a second independent reason for default besides overindebtedness. Inability to pay is triggered by rational behavior of incompletely informed outsiders. The firm needs to raise additional cash via secondary equity offerings in order to service it's coupon payments. Underpricing of secondary equity offerings is explained as necessary for these offerings to be successful. In addition to Duffie/Lando (2001) we find that the liquidity risk has a strong impact on the current firm value and the optimal leverage. Credit spreads of debt in the primary market depend on the degree of liquidity risk. They can be lower or higher than in case without liquidity risk.

1 Introduction

The valuation of defaultable claims has been a widely discussed topic in finance research. Starting with Black/Scholes (1973), Merton (1974), and Black/Cox (1976) the structural approach models the dynamics of the asset value and assumes default when some exogenously or endogenously specified default boundary is reached. Generalizations concerning interest rate risk or deviations from absolute priority have been provided, could, however, not solve the well known problem of short term risklessness evident in all traditional structural models under complete information. The continuity of the asset process results in the predictability of the default event, which in turn leads to riskless returns over short periods of time. Since for realistic parameter values this risklessness can show for maturities of up to a couple of months[1], structural models are not very useful for practical purposes, despite their theoretical elegance and sound economic explanation of the default process.

While in traditional structural models the default intensity is zero, reduced form models assume a positive default intensity and are thereby able to generate realistic credit risk premiums for all maturities. Put forward by Jarrow/Turnbull (1995), Jarrow/Lando/Turnbull (1997), Duffie/Singleton (1999) and others, default in this model class is not linked to economic parameters but rather occurs according to some exogenous hazard rate process[2].

Recently several authors have attempted to combine the favorable characteristics of both model classes without having to accept the respective disadvantages. Examples of these hybrid models are the frameworks by Madan/Unal (2000), Duffie/Lando (2001), Giesecke (2001), or Cathcart/El-Jahel (2002). Their common goal is to generate realistic credit premiums for short term maturities without having to give up the economic reasoning of default which is inherent in structural models.

Our contribution to literature stems from generalizing the framework by Duffie/Lando (2001) in several aspects. Duffie/Lando (2001) were able to show that a positive default intensity exists in a structural model, once information about the value of the firm's assets is no longer publicly available. In their framework, however, decisions concerning the firm and coupon payments to debtholders are exclusively made by insiders, who are aware of the firm's asset value at all times. As such, the firm's ability to service it's debt, the value of the levered firm and it's optimal capital structure are not affected by incomplete information. At the time of issuance, shareholders and debtholders in the Duffie/Lando (2001) model face the same risk as in comparable models with complete information, such as e.g. in the Leland (1994) framework. Since the risk is the same, prices of defaultable claims against the firm do not differ and incomplete information has no impact.

In our framework we endogenize inability to pay as a second independent

[1] See e.g. Giesecke (2002).

[2] For a thorough overview of traditional structural and reduced form models see Uhrig-Homburg (2002).

default trigger besides overindebtedness. Inability to pay is caused by rational decisions of outsiders faced with incomplete information concerning the firm's asset value. Outsiders are asked to provide additional equity as the firms needs to raise money to service it's debt payments. They than decide whether or not to proceed to a costly due dilligence based on incomplete information. If they decide not to proceed, the firm defaults due to inability to pay for it's interest obligations, regardless of the true value of the firm.

As the actions of incompletely informed agents have an influence on the default probability of the firm, the values of debt and equity are effected by the noise contained in the information that triggers the agent's actions. The firm's optimal capital structure and it's levered value are thus dependent on the degree of noise contained in publicly available information. We find that for a given leverage, risk premiums are always higher for firms facing incomplete information, due to the additional risk of default. Given optimal leverage, however, firms facing incomplete information exhibit *lower* credit spreads as a result of dramatically lower optimal debt levels. Other that that we find the influence of incomplete information on risk spreads to be limited, since investors turn to other sources of information once the noise contained in balance sheet reports increases too much.

The remainder of this article is structured as follows. Section 2 gives a detailed description of our model framework and describes the valuation process for defaultable claims as well as the derivation of the optimal capital structure. Section 3 provides a comparative static analysis of the model, comparing cases of complete and incomplete information as well as primary and secondary market valuations. Section 4 concludes.

2 Model framework

2.1 Independent variable and accounting information

Consider an all equity firm whose after tax asset value V_t follows a geometric Brownian motion

$$dV_t = \mu V_t \, dt + \sigma V_t \, dW_t \tag{1}$$

whereby μ and σ denote the instantaneous rate of return and the volatility of return, respectively. W is a standard Brownian motion, t denotes an arbitrary point in time. V_t is interpreted as the present value at time t of all expected future after tax cash flows from the firm's assets and considered to be non traded[3]. It is assumed that information about the value of V_t is not available publicly. For the numerical computations in section 3 we will use a binomial version of (1).

At $t = 0$ the firm faces the option to raise debt capital by a public offering in order to be able to profit from the tax advantage. Proceeds from the debt sale are supposed to be paid out to shareholders as a special dividend. Interest is paid on debt at discrete times and financed by additional equity issues, such that the stochastic process of V_t is unaffected by the financial structure of the firm. Should the firm be unable to raise sufficient capital to service it's debt payments, it goes bankrupt and incurs bankruptcy costs such that remaining assets to the amount of $\rho \cdot V_\tau$ are distributed among claimants on the basis of strict absolute priority for debtholders. ρ hereby denotes the percentage of recovery and V_τ denotes the value of the assets at the time of bankruptcy τ.

In order to raise additional equity to service due interest payments the company has to publish balance sheet information at every coupon date. Due to various accounting rules and restrictions the published balance sheet value B_t and the market value V_t of the assets need not be the same. The market value of the assets V_t is given as the unique solution of the SDE (1)

$$V_t = e^{Z(t)} \tag{2}$$

with

$$Z(t) = Z_0 + mt + \sigma W_t \tag{3}$$

and

$$m = \mu - \frac{\sigma^2}{2} \; . \tag{4}$$

Given the information at $t = 0$, the firm value $\tilde{B}_t$ reported in the balance sheet is also a random variable. We assume that $\tilde{B}_t$ is given by

$$\tilde{B}_t = e^{\tilde{Y}(t)} \; , \tag{5}$$

where $\tilde{Y}_t$ consists of two components,

$$\tilde{Y}_t = Z(t) + \tilde{U}(t) \; , \tag{6}$$

[3]Since the value of the unlevered firm is a non traded asset, parallel co-existence of both the optimally levered and the all equity firm does not pose problems concerning possible arbitrage opportunities.

the return $Z(t)$ of the firm value V_t and the normally distributed noisy deviation $\tilde{U}(t)$,

$$\tilde{U}(t) \sim N(-\frac{\kappa^2}{2}, \kappa) \ . \tag{7}$$

$\tilde{U}(t)$ and $Z(t)$ are assumed to be independent and κ denotes the standard deviation of $\tilde{U}(t)$. It is straightforward to see that the accounting based firm value B_t is an unconditional and conditinal unbiased report of V_t since

$$E_0\left(\tilde{B}_t\right) = E\left(e^{Z(t)}\right) \cdot E\left(\tilde{U}(t)\right) = V_t \ , \tag{8}$$

$$E_t\left[\tilde{B}_t \middle| Z(t)\right] = e^{Z(t)} \cdot E\left[e^{\tilde{U}(t)}\right] = V_t \ . \tag{9}$$

2.2 Agents

The model involves three groups of risk-neutral agents, who can be further categorized into insiders and outsiders. Insiders are at all times aware of the true value of the firm's assets V_t, while outsiders knowledge is limited to public information.

Managers/shareholders are insiders and try to maximize their wealth. They are assumed to be unable to convincingly signal their knowledge of the firm's asset value to outside parties and restricted from participating in secondary equity offerings due to a lack of liquid assets. Managers/shareholders are also by assumption precluded from trading in the secondary market for corporate bonds, due e.g. to insider-trading regulations. Insider information, therefore, is not revealed in bond prices.

Since managers/shareholders are unable to participate in secondary equity offerings, new shares are offered to potential investors, who are outsiders. These potential outside investors cannot directly observe the asset value, but rather have the opportunity to learn V_t by performing a special audit of the firm at cost C. It is assumed that potential investors will not buy new shares of the firm without knowing the exact value of it's assets[4]. Upon investing in the new shares of the firm potential investors turn into managers/shareholders and thus become insiders.

[4]It can be shown numerically that, except for unrealistically high auditing costs, it is profitable for potential investors to first audit and then buy new shares.

The third group of agents are traders in the primary market for corporate bonds. They are also outsiders and assumed to be precluded from participating in the firm's secondary equity offerings. In case of bankruptcy bondholder's claims D are serviced on the basis of strict absolute priority.

2.3 Public and private information

A very important aspect of this model is to know what kind of information both insiders and outsiders possess at each point in time. It is assumed that at $t = 0$ no difference exists between public and private information. The initial asset value V_0, as well as the parameter values μ and σ of equation (1) are known, such that all possible future asset values V_t and their respective unconditional probabilities of occurrence can be derived from public information. It is further assumed that relationships (5)-(7) are known such that the conditional distribution of possible balance sheet values $f(B_t|V_t)$ is public information.

As time moves on to $t > 0$ anything that used to be known in $t = 0$ remains known, i.e. agents do not "forget" information they once possessed. The difference, however, is that information about the true state of the asset value $V_{t>o}$ is no longer publicly available but rather private information. Outsiders thus know which state the asset value could be in and derive a probability distribution conditional on their public information, namely the last known asset value and the dynamics of V_t from equation (1).

At the time of the coupon payment the firm reports a balance sheet according to relationships (5)-(7). While this accounting information is noisy and does not allow outsiders to pinpoint an asset value with certainty, it provides them with an opportunity to update their public information. Figure 1 gives an example of this information update for discrete firm values.

In this example outsiders know that there are three possible assets values in $t = 2$. Having received the accounting information B_2 they calculate the conditional probability of the true state being V_{ti} as

$$P(V_{2i}|B_2) = \frac{P(V_{2i} \cap B_2)}{P(B_2)} = \frac{f(B_2|V_{2i}) \cdot P(V_{2i})}{\sum_{j=0}^{2} f(B_2|V_{2j}) \cdot P(V_{2j})}. \quad (10)$$

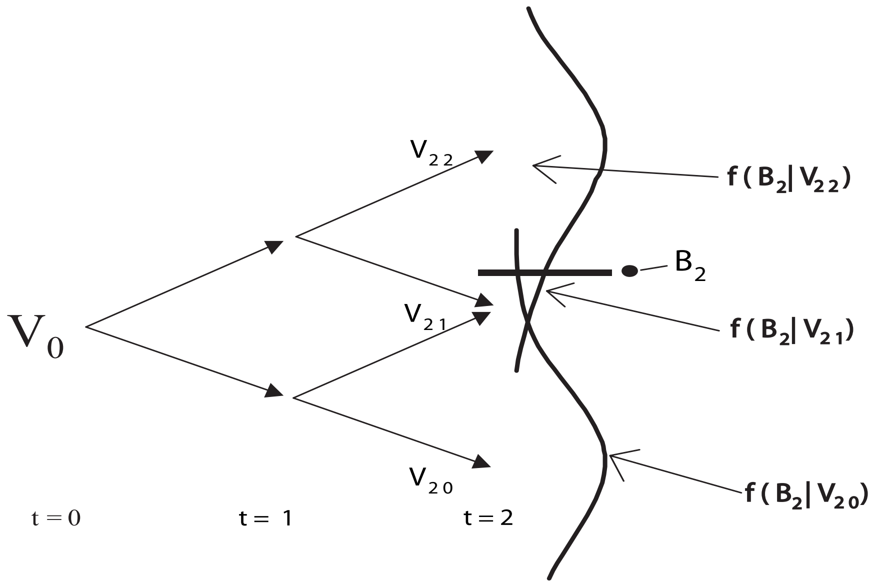

Figure 1: Conditioning public information on a balance sheet reported in t=2

This public information updated with the accounting value of the firm's assets is the basis on which outsiders make their decision regarding participation in the firm's concurrent secondary equity offering.

2.4 Secondary equity offering

In order to finance it's interest payments the firm has to place a secondary equity offering with outside investors at every coupon date. Figure 2 gives an overview of the secondary equity placement process. Starting point are the potential equity investors, who are willing to subscribe for new shares as long as it is a worthwhile investment. Since they are risk-neutral, they decide on the basis of the expected value criterion whether or not to invest. As they do not, by assumption, invest without knowing the true value of the assets, their first decision is to whether or not spend the amount C necessary to audit the firm. The potential investor's decision is based on their public information and takes into account all possible states of asset values and the corresponding gains and losses that foreseeably occur in each state as a result of rational behavior. If the potential investors decide not to subscribe

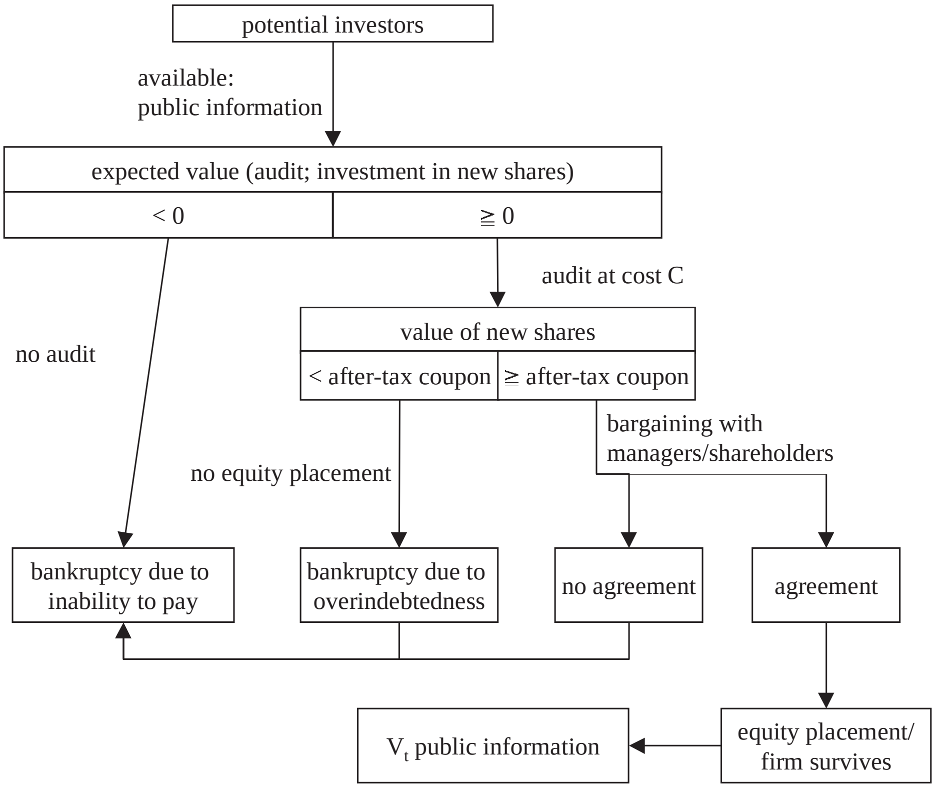

Figure 2: Overview of secondary equity placement process

for new shares, the firm is unable to raise the capital necessary to service it's interest payments and goes bankrupt without necessarily having to be overindebted.

If the potential investors decide to audit the firm, they learn the true asset value V_t and are thus equally well informed as the managers/shareholders. Knowing the true state of the firm, the potential investors can calculate the value of the firm's equity and thus know whether or not an investment is potentially rewarding. Should the value of the firm's equity after the coupon payment be less than the after tax coupon payment itself, the firm would be overindebted after an equity financed coupon[5]. In this case the managers/shareholders have nothing to offer to the investors to make it a profitable investment for them. Therefore, the potential investors will not participate in the equity offering even after the costly audit and the firm will be unable to pay it's interest obligations as well as overindebted.

Should the asset value justify a secondary equity offering, i.e. the firm is not overindebted, it still remains how much ownership the managers/shareholders are willing to give up to the investors. Managers/shareholders and potential investors thus enter into a bargaining process about the share of the firm that investors receive in return for their investment. The potential investor's bargaining power η is assumed to be public information.

If both parties agree, the placement succeeds and the firm survives. The new ownership is entered on the Commercial Register and since the amount necessary to service the interest payment is publicly known, the true asset value can be deducted and thus becomes public information. Should the parties not agree, the placement does not succeed and the firm goes bankrupt due to inability to pay for it's interest obligations.

2.5 Bargaining process

Managers and potential investors bargain over the share of the firm that investors receive in exchange for their equity investment. As can be seen in figure 2, at the time of the bargaining process the true asset value is known to both parties and auditing cost are to be considered sunk. The investment necessary to save the firm from bankruptcy equals the after tax coupon payment $(1 - s) \cdot K$, where K is the nominal coupon and s is the

[5] This is equal to the endogenous default condition in the sense of Black/Cox (1976) or Leland (1994).

	offering successful	offering not successful
managers/shareholders	$(1-\Theta_t)\cdot E_t^+$	$\max(\rho\cdot V_t-(D+K),0)$
potential investors	$\Theta_t\cdot E_t^+-(1-s)\cdot K$	0

Table 1: Bargaining outcomes

corporate tax rate. The opportunity to negotiate exists, since by agreeing the parties can save a deadweight loss to the amount of the bankruptcy costs $(1-\rho)\cdot V_t$ and the otherwise lost tax shield of debt.

The parties' possible strategies are to either agree to the secondary equity placement and a share Θ_t of the firm that the potential investors receive in return for their investment, or not to agree. Θ_t thereby depends on the true value of the firm. If both parties agree, the firm survives and Θ_t of the firm's equity go to the new shareholders. The value of the investment to the new shareholders thus equals their share in the company Θ_t times the value of the firm's equity after the coupon payment E_t^+ minus the price of the investment $(1-s)\cdot K$. If the parties do not agree, the company goes bankrupt and managers/shareholders receive what is left of the assets after bankruptcy cost and debtors claims consisting of the nominal debt value D and the accrued coupon K have been served. Potential investors receive nothing. Table 1 gives a summary of the possible bargaining results.

Potential investors and managers/shareholders face an asymmetric Nash-bargaining problem:

$$\Theta_t=\arg\max_{0\leq\Theta\leq 1}\left[\Theta\cdot E_t^+-(1-s)\cdot K-0\right]^{\eta}\cdot \left[(1-\Theta)\cdot E_t^+-\max(\rho\cdot V_t-(D+K),0)\right]^{1-\eta} \quad . \tag{11}$$

The solution of (11) is given by

$$\Theta_t=\min\left[\eta+\frac{(1-s)\cdot K\cdot(1-\eta)}{E_t^+}\;,\; \frac{(1-s)\cdot K+\eta\cdot(E_t^+ + D-(1-s)\cdot K-\rho\cdot V_\tau}{E_t^+}\right]. \tag{12}$$

The deadweight loss which is avoided in case of an agreement is thus distributed among the parties according to their respective bargaining power.

Since all agents in the model behave rationally and potential investors know their bargaining power, they can anticipate the overall gains and losses they will incur in each possible state of the asset value. They decide to audit the firm when the certain auditing costs are offset by an expected gain from participating in the secondary offering of at least an equal amount.

2.6 Risk analysis

In order to value defaultable claims and to derive an optimal capital structure in this framework it is important to understand what drives default risk. Default occurs as a consequence of either inability to pay or overindebtedness, both of which are reasons to file for bankruptcy. Default due to overindebtedness is a common feature in structural models and occurs as the asset value crosses an either exogenously[6] or endogenously[7] specified default boundary. Since in most structural models cash necessary for interest payments is raised via secondary equity offerings, overindebted firms are usually also unable to pay for their interest obligations. However, inability to pay as an independent reason for default is difficult to explain in frictionless markets, since firms that are not overindebted should be able to raise cash to service their interest payments. Kim/Ramaswamy/Sundaresan (1993) model inability to pay as an independent default trigger, but are unable to account for this problem[8]. Our framework allows for an independent default due to inability to pay as well as overindebtedness, since it relaxes the assumption of frictionless markets by introducing incomplete information.

The risk of default due to overindebtedness is driven by the dynamics of the asset value process as described by equation (1). Overindebtedness is measured in market values and is in principle equivalent to the endogenous default in other structural models such as Black/Cox (1976) or Leland (1994, 1998). The only difference is that interest is only paid at discrete times and overindebtedness can thus only become evident at these coupon dates.

Inability to pay occurs if the potential investors come to the conclusion that the expected value of the secondary equity offering does not warrant spending the amount C necessary to audit the firm. It, too, can only become evident at the coupon dates. Potential investors derive their public information as described by equation 10, which obviously depends on the stochastic balance sheet value $\tilde{B}_t$. It is therefore evident that the risk of default due

[6] See e.g. Merton (1974) or Longstaff/Schwartz (1995).
[7] See e.g. Black/Cox (1976) or Leland (1994, 1998).
[8] See Uhrig-Homburg (2002), pp. 33-43.

to inability to pay is driven by two sources, namely the dynamics of the asset value process and the noise contained in the balance sheet information. These two sources thus have to be considered when valuing defaultable claims written on the value of the firm.

2.7 Valuation

Valuation in this framework is done numerically, using binomial trees. The nodes i of this tree at time t will be denoted by ti. The values of the claims against the firm thus have to be calculated recursively, conditional on the true asset value. Since, however, the dynamics of the asset value are not the only source of risk, valuation also has to consider the factors influencing public information in the coupon dates, namely the last known asset value and the balance sheet information.

Having to consider the last known asset value makes the value of the firm path dependent. Since, however, as soon as the true asset value is publicly known the history of asset values becomes irrelevant, path dependencies result only between two successive coupon dates and can be accounted for numerically at acceptable computational costs.

More difficult to consider is the random balance sheet information which is published at every coupon date. It is important to see that the balance sheet value has an impact only on the outside investor's decision of whether or not to audit the firm. It neither has an impact on whether or not the firm defaults due to overindebtedness nor on the conditions Θ_{ti} of the secondary equity offering. If the firm is overindebted according to market valuation, an excellent balance sheet value might lead the potential investors to audit the firm. They will, however, not invest as they then learn about the true value of the assets and the firm defaults. A low balance sheet value will lead the potential investors not to audit and the firm will also default. In this case it is irrelevant for valuation purposes whether or not an audit has been performed, since the auditing cost are spent by the potential (outside) investors and default occurs in either case. Once managers/shareholders and potential investors get to bargain over the conditions Θ_{ti} of the secondary equity offering, the true asset value is known to both parties and the balance sheet value is also irrelevant.

In terms of default the balance sheet value therefore only has an impact if the firm is not overindebted. It governs whether or not the potential investors decide to audit the firm and thus triggers default in the case where the firm is unable to pay but not overindebted.

Since higher balance sheet values lead to higher expected asset values while higher asset values lead to more profitable bargaining results for the potential investors and auditing cost are fixed, the expected value of auditing the firm rises monotonically with the balance sheet value B_t. Thus there exists one critical balance sheet value B_t^* for which the expected value of the potential investor's strategy "audit and invest optimally" equals zero. The probability of default due to inability to pay (IP) conditional on the asset value V_t is therefore given as

$$P(IP|\, V_{ti}) = \int_0^{B_t^*} f[B_t|\, V_{ti}]. \tag{13}$$

It is now possible to value the debt and equity claim conditional on the asset value V_t of the firm. Overindebtedness only depends on the asset value and can thus be determined at any time by recursive calculation. If the firm is overindebted, $\rho \cdot V_{ti}$ is distributed to the debtholders on the basis of strict absolute priority. If the firm is not overindebted, the probability of default due to inability to pay is calculated as described by equation 13. The values of the claims in case of survival of the firm are calculated recursively, the values in case of default are determined by the strict absolute priority of the debt claim and the bankruptcy costs. Since all agents in the model are risk-neutral, the value of the claims immediately before the balance sheet is published equals the sum of their respective default and survival values, weighted with the probability of default due to inability to pay and survival, respectively.

To calculate the values of debt and equity by backward iteration, we need an assumption regarding the value of the firm at time T of the maturity of the debt claim. We assume that the firm needs to raise equity in order to cover the last coupon payment and after that is able to liquidate it's assets without cost in order to repay the principle of the bond. With this assumption we follow Brennan/Schwartz (1978) who assume that the value of the levered firm equals the value of the unlevered firm at maturity of debt. The value of the total equity immediately after the last coupon payment E_T^+ therefore amounts to

$$E_T^+(V_{Ti}) = \max(V_{Ti} - D, 0)\ . \tag{14}$$

When calculating the value of equity, however, it has to be considered that the initial owner's share in the company is systematically diluted by the secondary equity offerings. This is accounted for by subtracting the value of the new shareholder's stake in the equity $\Theta_{Ti} \cdot E_T^+(V_{Ti})$ from the value of total equity after the last offering $E_T^+(V_{Ti})$. Therefore, the value of the "old" shares $(1-\Theta_{Ti}) \cdot E_T^+(V_{Ti})$ must also be used in the backward iterarion. We denote by E_T the value of the equity stake of the "old" investors at the time the true asset value is revealed

$$E_T(V_{Ti}) = \begin{cases} (1-\Theta_{Ti}) \cdot E_T^+(V_{Ti}), & \text{for } E_T^+(V_{Ti}) \geq (1-s) \cdot K \\ \max\left(\rho \cdot V_{Ti} - (D+K), 0\right), & \text{else.} \end{cases} \tag{15}$$

The value of the equity claim to the "old" shareholders immediately before maturity E_T^-, which considers the probability of default due to inability to pay, is given as

$$E_T^-(V_{Ti}) = P\left(IP\,|V_{Ti}\right) \cdot \max\left(\rho \cdot V_{Ti} - (D+K), 0\right) + \left(1 - P\left(IP\,|V_{Ti}\right)\right) \cdot E_T(V_{Ti}). \tag{16}$$

This accounts for the equity offering under incomplete information. Since new information is revealed only in coupon dates,

$$E_t^-(V_{ti}) = E_t^+(V_{ti}) \tag{17}$$

holds for all times different from coupon dates and the value of the equity claim can be carried backwards in the binomial tree by discounting at the riskless rate

$$E_t^+(V_{ti}) = \left[p \cdot E_{t+1}^-(V_{t+1,i+1}) + (1-p) \cdot E_{t+1}^-(V_{t+1,i})\right] \cdot e^{-r \cdot \Delta t}. \tag{18}$$

p hereby denotes the riskneutral probability to move up the binomial tree from state V_{ti} to state $V_{t+1,i+1}$.

At an arbitrary coupon date the value of total equity immediately after the coupon payment is given by

$$E_t^+(V_{ti}) = \left[p \cdot E_{t+1}^-(V_{t+1,i+1}) + (1-p) \cdot E_{t+1}^-(V_{t+1,i})\right] \cdot e^{-r \cdot \Delta t}. \tag{19}$$

In line with equations (15) and (16) the value of equity to the "old" shareholders at the time of dissemination of the true asset value and immediately before the coupon payment are given by

$$E_t(V_{ti}) = \begin{cases} (1-\Theta_{ti}) \cdot E_t^+(V_{ti}), & \text{for } E_t^+(V_{ti}) \geq (1-s) \cdot K \\ \max\left(\rho \cdot V_{ti} - (D+K), 0\right), & \text{else} \end{cases} \tag{20}$$

and

$$E_t^-(V_{ti}) = P\left(IP\left|V_{ti}\right.\right) \cdot \max\left(\rho \cdot V_{ti} - (D+K), 0\right) + \left(1 - P\left(IP\left|V_{ti}\right.\right)\right) \cdot E_t(V_{ti}), \tag{21}$$

respectively. This procedure continues until the initial equity value in $t = 0$ has been found.

Since the firm defaults when it is overindebted, the market value of the debt claim MD in a given state depends on the value of equity. The value of the debt claim at maturity T is given by

$$MD_T(V_{Ti}) = \begin{cases} D + K, & \text{for } E_T^+(V_{Ti}) \geq (1-s) \cdot K \\ \rho \cdot V_{Ti}, & \text{else.} \end{cases} \tag{22}$$

Immediately before the last coupon payment, the firm is still prone to default due to inability to pay and the debt value is given by

$$MD_T^-(V_{Ti}) = P\left(IP\left|V_{Ti}\right.\right) \cdot \min\left(D+K, \rho \cdot V_{Ti}\right) + \left(1 - P\left(IP\left|V_{Ti}\right.\right)\right) \cdot MD_T(V_{Ti}). \tag{23}$$

Since between coupon payments no new information reaches the market,

$$MD_t^-(V_{ti}) = MD_t^+(V_{ti}) \tag{24}$$

holds for all times different from coupon dates and the value of the debt claim at an arbitrary point in time can be calculated by backward iteration

$$MD_t^+(V_{ti}) = \left[p \cdot MD_{t+1}^-(V_{t+1,i+1}) + (1-p) \cdot MD_{t+1}^-(V_{t+1,i})\right] \cdot e^{-r \cdot \Delta t}. \tag{25}$$

The value of the debt claim immediately after a coupon date is conditional on the firms survival and derived through backward iteration

$$MD_t^+(V_{ti}) = \left[p \cdot MD_{t+1}^-(V_{t+1,i+1}) + (1-p) \cdot MD_{t+1}^-(V_{t+1,i})\right] \cdot e^{-r \cdot \Delta t}. \tag{26}$$

The debt value at the coupon date depends on whether the firm defaults due to overindebtedness

$$MD_t(V_{ti}) = \begin{cases} MD_t^+ + K, & \text{for } E_t^+ \geq (1-s) \cdot K \\ \min\left(D+K, \rho \cdot V_{ti}\right), & \text{else.} \end{cases} \tag{27}$$

The debt value immediately before the coupon date incorporates the probability of default due to inability to pay and is given as

$$MD_t^-(V_{ti}) = P\left(IP\left|V_{ti}\right.\right) \cdot \min\left(D+K, \rho \cdot V_{ti}\right) + \left(1 - P\left(IP\left|V_{ti}\right.\right)\right) \cdot MD_t(V_{ti}). \tag{28}$$

This procedure continues until the initial debt value in $t = 0$ has been found.

2.8 Optimal capital structure

It is assumed that the "original" managers/shareholders of the unlevered firm choose a capital structure in $t = 0$ which is not actively changed afterwards. Since they try to maximize their personal wealth and the proceeds of the debt sale are by assumption distributed as a special dividend, the optimal capital structure is found by maximizing the market value of the firm which equals the sum of the debt and the equity value.

As in other structural models with optimal capital structure[9] the values of the levered and the unlevered firm differ by the present value of bankruptcy costs and the debt tax shield. Contrary to these models, however, secondary equity offerings in this framework are systematically underpriced. They have to be, since otherwise the potential investors would not have an incentive to audit the firm and default due to inability to pay would be inevitable. Underpricing to the amount of $\Theta_{ti} \cdot E_t^+(V_{ti}) - (1 - s) \cdot K$ therefore occurs in every coupon date. The present value of expected underpricing further reduces the value of the levered firm to it's original owners and thus has to be considered when making the capital structure decision in $t = 0$.

3 Comparative static analysis

The following comparative static analysis will help to better understand the model and show some interesting results and implications. Due to the necessary use of numerical procedures, however, we cannot claim generality and from time to time we have to smooth resulting graphs for better presentability. Results will be generated by varying one parameter of the model while holding the others constant. The parameters of the base case mostly correspond to those of Leland (2002) and Huang/Huang (2002) and were chosen as follows.

The asset volatility σ was taken to be 23% in the base case scenario. Leland (2002) shows that the asset volatility turns out to be approximately 23% if we assume that individual stocks[10] have an average volatility of 35% and firms are on average 35% debt financed. The riskless rate of return was

[9] See e.g. Kraus/Litzenberger (1973), Leland (1994) or Hilberink/Rogers (2002).

[10] Assuming a correlation of 0.2 between individual stocks, this assumption implies a risk of 20% for well diversified equity portfolios. This risk equals the long term S&P500 index volatility.

chosen to be 8% which is close to the historical average of Treasury rates during 1973-1998[11]. We assume a flat term structure of interest rates and abstract from interest rate risk. The recovery rate is assumed to be 70% which is equivalent to bankruptcy costs of 30% as in Leland (2002). The corporate tax rate is taken to be 35% and we assume two coupon payments per year. Parameter values for the bargaining power of the potential investors, auditing costs and the degree of incomplete information can not be derived from empirical findings and therefore have to be fixed reasonably. We assume the bargaining power of the potential investors to be 1%, which is equivalent to saying that they capture 1% of the deadweight loss which is avoided by their decision to invest. Auditing costs are assumed to equal 0.01% of the unlevered firm value at $t = 0$. The degree of incomplete information is governed by the standard deviation κ of the disturbance term $\tilde{U}(t)$ and taken to be 10% in the base case. In what follows, we will assume that the firm issues debt at par value, thereby eliminating difficulties concerning calculation of the proper tax shield for bonds issued at a discount or a premium.

In varying parameters of the model for a comparative static analysis we will consider two different scenarios. Both scenarios concern the firm at time $t = 0$. In this situation the firm value V_0 is publicly known and the firm is all equity financed. Our first scenario will consider fixed debt levels which are given exogenously. As such, managers do not have the opportunity to actively influence the firm's debt level as parameters are varied. In our second scenario, managers decide optimally on how much debt to raise in order to maximize their wealth. They therefore react to different parameter settings by adapting their financing policy for the firm. Comparative static analysis in both settings will look at the credit spreads of the issued debt. Results will be compared to an otherwise identical firm with complete information.

3.1 Comparative statics for fixed debt levels

In this part of our comparative static analysis we will treat the nominal debt level as given. We assume that the firm issues debt with a face value to the amount of 50% of it's unlevered value. Since we also assume that debt is issued at par, the bond's face value is equal to it's value in the primary market.

[11]See. Huang/Huang (2002), p.20.

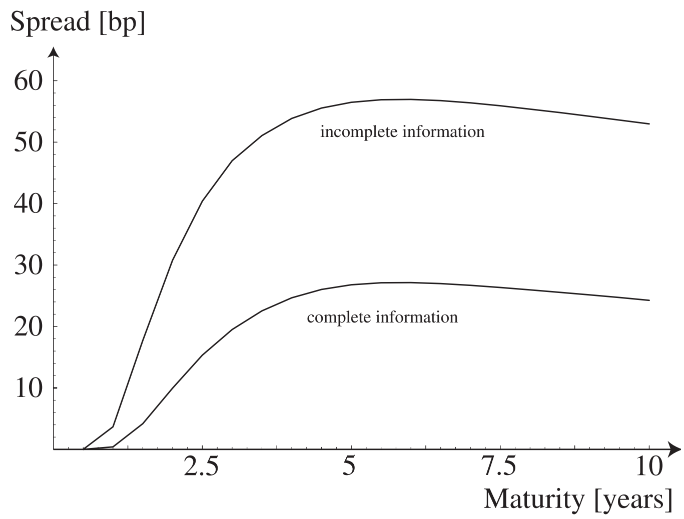

Figure 3: Spread vs. time to maturity, fixed debt level

3.1.1 Term structure of credit spreads

The first parameter we will vary is the time to maturity T. For the base case scenario with complete information one expects to see a hump shaped term structure with credit spreads going to zero for short maturities. This is a well documented phenomenon of structural models and has led to many critiques regarding their empirical suitability.

As can be seen from Figure 3, incomplete information cannot avoid short-term risklessness in the primary market. Since V_0 is known, agents know that immediate default is impossible. Due to the discrete time step Δt selected, credit spreads are not noticeably different from zero in both models even for maturities up to $1/2$ year. Only for longer maturities we see positive credit spreads, while the spreads are strictly larger for the model with incomplete information. This should be no surprise, since the uncertainty that results from incomplete information triggers default due to inability to pay which cannot happen in the model with complete information and thus comes as an additional source of risk. One can see, however, that the risk premium due to incomplete information rises with increasing maturity up to a certain level where it remains constant. This leads the two models to have almost parallel term structures of credit spreads for longer maturities.

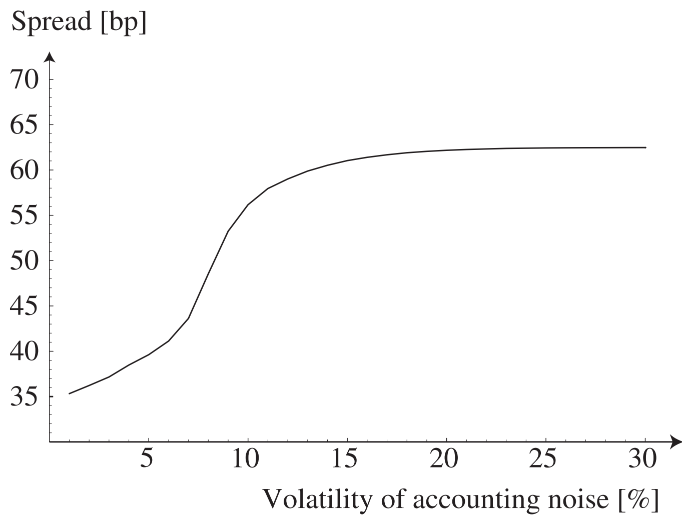

Figure 4: Spread vs. accounting noise, fixed debt level

3.1.2 Variation of the degree of informational incompleteness

The main friction we introduced into our model is incomplete information, which obviously cannot be measured empirically. As such it is interesting to see what happens to our results as we change our assumptions regarding the level of noise contained in the firm's balance sheet information. As a reference we take a bond with five years to maturity from our base case scenario, which exhibits a credit spread of 26.8 [bp] with complete information.

Several aspects regarding the comparative static analysis in Figure 4 are noteworthy. Introducing incomplete information results in additional defaults due to inability to pay that occur in states where the firm is not overindebted. As such, the credit spread of the five year bond jumps to 35.3 [bp] as we increase the noise volatility from 0% to 1%. While we keep increasing uncertainty, investors demand compensation at an increasing rate until the curve starts to level off and converges to approximately 62 [bp] for noise volatility over 25%.

The mechanism of this effect becomes clear as one considers how accounting noise affects the default probability due to inability to pay. At the time when potential investors make their decision whether or not to audit the firm, they determine if the firm will default due to inability to pay. This is, since they will invest as long as the investment is worthwhile (i.e. the firm is not overindebted) and otherwise the firm will default due to overindebtedness anyway. They make their decision to audit on the basis of public information which at that time consists of the last known asset value, the dynamics of the firm value process and balance sheet information. The more noise, however, is contained in the balance sheet information, the less valuable it becomes. In an extreme case where accounting noise goes to infinity, the balance sheet value not longer contains information and thus becomes worthless. In this case, public information concerning the asset value of the firm on the coupon date is based solely on the last known firm value and the dynamics of the asset value process. It is therefore obvious that the probability of default due to inability to pay should converge to the case where no balance sheet information is available. Since default probabilities due to overindebtedness are not concerned, credit spreads level off.

3.1.3 Variation of the recovery rate

In our model we follow the common recovery of firm value assumption which states that a certain percentage (recovery rate) ρ of the firm value immediately before default V_τ is distributed among claimholders on the basis of strict absolute priority. Since default constitutes a risk for debt investors only if the firm's assets are not sufficient to satisfy their claims, the higher the recovery rate the less investors stand to lose. One would therefore expect to see declining credit spreads with rising recovery rates. As can be seen in Figure 5 this is the case for both models with complete and incomplete information. Some other aspects, however, remain and are noteworthy.

Credit spreads in the model with incomplete information are both higher and react stronger to changes in the recovery rate. Both phenomenons are plausible once one considers that the probability of default is higher in the model with incomplete information due to the additional risk of default due to inability to pay. While the credit spreads in the model with complete information, however, exhibit a linear shape when recovery rates are varied, credit spreads with asymmetric information show a slightly convex shape which turns into concavity near 100% recovery.

Since under complete information default happens only due to overindebtedness and thus in states where equity holders do not expect to receive anything after the debtholders claims have been considered, changing recovery rates have no influence on the value of equity. As such, the barrier where

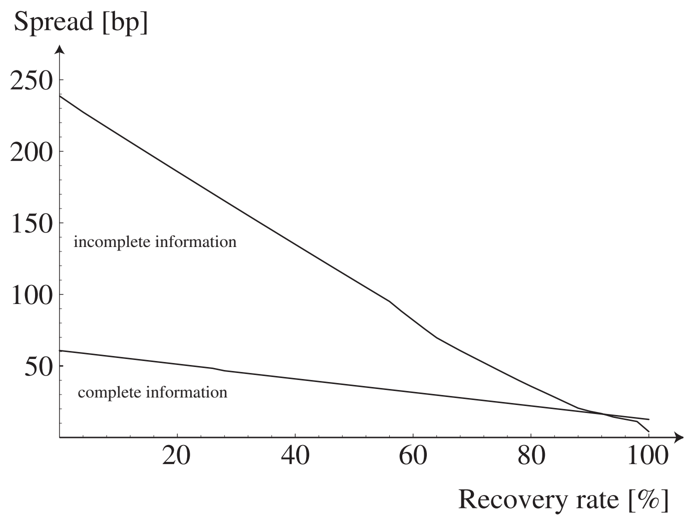

Figure 5: Spread vs. recovery rate, fixed debt level

managers endogenously trigger default is also unaffected. Thus rising recovery rates do not change default probabilities but what is left of the firm in case of default. This leads to the linear behavior in the model with complete information.

In the model with incomplete information default due to inability to pay happens in states where the firm is not overindebted. In such states equity holders have a reason to expect residual payments if the recovery rate is high enough. Therefore rising recovery rates not only lead to more value left over for debt holders in case of default but also to lower default probabilities as equity also gains from rising recovery rates. This causes the spread curve to become convex.

Rising recovery rates, however, also have an indirect effect on the probability of default due to inability to pay. The reason is that old shareholders and potential investors while bargaining over the conditions of the secondary equity offering effectively bargain over the share of the company that new shareholders are to receive in return for their investment. At the time of the negotiation both parties are aware of the true asset value and thus know what they are to gain or to loose in case they do not agree. Deadweight loss that can be avoided by agreement and thus stands to be distributed among the parties consists of two parts, namely bankruptcy costs and expected future tax shield, both of which are lost should the firm default. Rising recovery rates, however, move the "old" shareholders into a stronger position to negotiate since a full recovery of the assets, for example, leaves them with only the tax shield to lose. Potential investors therefore anticipate that there is not much to negotiate about after they get to know the true asset value of the firm. Since they cannot expect to recover their auditing costs, they are likely not to audit and the firm defaults.

High recovery rates therefore on the one hand lead to high default probabilities due to inability to pay while on the other hand they leave a lot of assets to distribute among claimants once the firm defaults. Both effects lead to almost riskless rates, since debt holders can be reasonably sure to recover their entire claim in case of bankruptcy. This argument does not hold for the case of complete information, however, which is the reason that risk premiums are lower for firms with incomplete information. Full recovery does not lead to riskless debt in the case of complete information since managers decide optimally when to trigger bankruptcy. The asset value at which default is endogenously triggered is well below the nominal value, as Leland (1994) shows in a more general setting. As such, debt does not become riskless since in case of default investors still suffer a loss, even though the then current asset value is fully recovered.

3.1.4 Variation of the auditing costs

The next parameter to be varied are auditing costs. For our base case scenario we choose auditing costs to be 0.01% of the unlevered firm value in $t = 0$. For a leverage ratio of 50% and two coupon dates per year, auditing costs amount to 0.5% of the amount invested by the new shareholders, assuming debt is issued at the riskless rate of 8%. Since risky debt is issued at par, relative auditing cost will be slightly less than 0.5% of the invested amount.

As a reference we again take a bond with five years to maturity from our base case scenario, which exhibits a credit spread of 26.8 [bp] with complete information. If we introduce incomplete information and assume auditing costs to be 0.001% of V_0 (i.e. 1/10 of the costs of our base case scenario), credit spreads jump to 55.9 [bp]. Increasing auditing costs tenfold to the level assumed in our base case scenario increases the credit spread to 56.6 [bp], which can hardly count as a big difference. Doubling auditing cost again to 0.02% increases the credit spread by 0.07 [bp], which again would not be considered a remarkable increase and leads us to conclude that results are

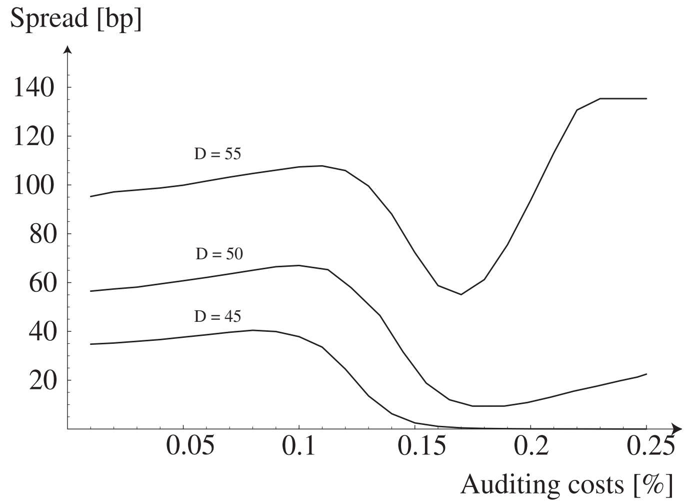

Figure 6: Spread vs. auditing costs, fixed debt level

not very sensitive to changes in the level of auditing costs.

Things start to look a little bit different, however, once we start to increase auditing cost more heavily, as can be seen from Figure 6.

We show credit spreads vs. auditing costs for three different levels of debt outstanding. As one would expect, credit spreads for given auditing costs increase with leverage since default becomes more probable. In line with these expectations we see spreads moderately increasing up to auditing costs of 0.1%, which is ten times the amount assumed in our base case scenario.

Increasing auditing cost further, however, leads to rising default probabilities even in states where the asset value is sufficient to satisfy bondholder's claims after bankruptcy costs have been considered. In these cases debt claims are fully repayed, sparing bondholders future risks and possible defaults in states where the asset value is not sufficient for repayment. This effect leads spreads to decline.

As we keep increasing auditing cost even further, however, default due to inability to pay at the first coupon date becomes a sure thing. As such, credit spreads for very high auditing costs reflect the expected results of a sure liquidation of the firm at the first coupon date. For debt values of 40 or less, it is almost certain that the asset value will be sufficiently high to fully satisfy the claims, which leads to riskless returns. For higher amounts of debt, however, bondholders have to take into account the possibility that asset values decline enough within the first half year to leave them with a loss. This is reflected in the positive spreads for high auditing cost, which quickly rise as higher amounts of outstanding debt are considered.

3.2 Comparative statics for optimal debt levels

The following comparative static analysis will no longer treat the amount of debt outstanding as exogenously given but will rather consider managers decision to optimally lever the firm in $t = 0$. As described in section 2.8, managers choose the amount of debt that maximizes the value of the firm, thereby maximizing initial shareholders personal wealth. As we vary different exogenous parameters, we will therefore implicitly consider different and changing capital structures.

3.2.1 Term structure of credit spreads

The first parameter to be varied is again the time to maturity T. We showed in section 3.1.1 that for fixed debt levels hump shaped term structures result. Furthermore, as expected, spreads for the model with incomplete information are larger than for the model with complete information due to the higher default probability. As we allow managers to chose optimal debt levels, however, we get very much different results as can be seen from Figure 7.

In this case spreads for the model with incomplete information turn out to be noticeably lower than in the case of complete information. The reason is that the levels of optimal leverage differ considerably. Whereas we find the optimal debt level in the base case of a 5 years to maturity bond to be 47% in the model with incomplete information, it increases to 74.5% in the model with complete information. The higher leverage overcompensates the additional default risk due to inability to pay and thus leads to higher spreads.

It remains an open question, however, why optimal debt levels differ so much. The reason, again, has to lie with the additional default probability due to inability to pay which is triggered by incomplete information. In the model with complete information increasing the amount of outstanding debt starting from level zero has two consequences for total firm value. The tradeoff between tax shield and bankruptcy costs determines the optimal capital structure at the point of leverage where the gain in tax shield from

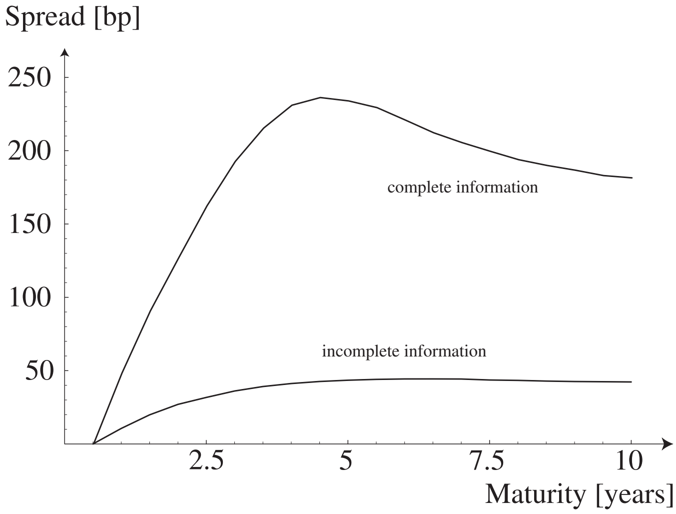

Figure 7: Spread vs. time to maturity, optimal leverage

the next unit of debt issued is offset by the loss from the increase in expected bankruptcy costs. In the case of complete information this exactly offsetting effect occurs at lower debt values for two reasons. First, the higher default probability in the model with incomplete information leads to both lower tax shields for given levels of debt and lower rates of increase when more debt is issued as compared to the model with complete information. The reason is that a positive effect (tax deductibility of a given coupon payment) which is equal in both cases is offset by a negative effect (loss of tax shield due to bankruptcy) which is different.

Second, absolute expected bankruptcy costs and the rate of increase are higher in the model with incomplete information. The reason is that absolute bankruptcy costs in all overindebted states of the asset value are equal for both models. Additional bankruptcy costs occur in states where the firm is unable to pay, which increases total bankruptcy cost for the model with incomplete information as compared to the case with complete information. Since higher leverage not only causes higher default probabilities due to overindebtedness but c.p. also raises the probability of default due to inability to pay via higher coupons that need to be financed, the rate of increase in expected bankruptcy costs is also higher for the model with incomplete information.

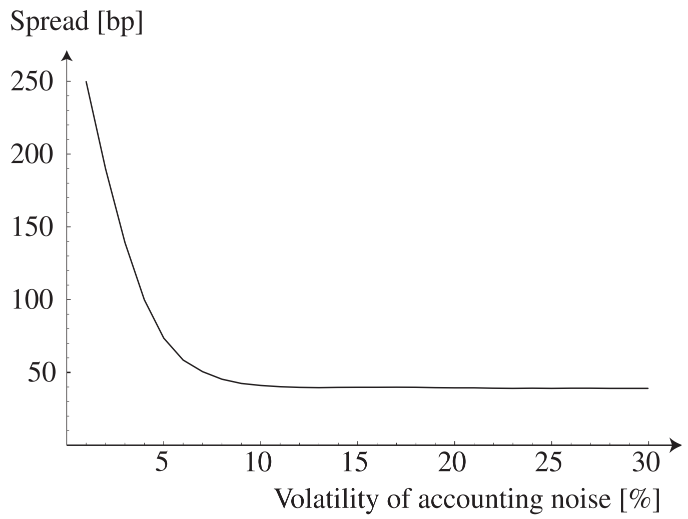

Figure 8: Spread vs. accounting noise, optimal leverage

Since with rising debt levels both the rate of increase in expected bankruptcy cost is higher and the rate of increase of tax shield is lower in the model with asymmetric information, the optimal capital structure turns out to exhibit significantly less leverage, which overall leads to lower credit spreads.

3.2.2 Variation of the degree of informational incompleteness

In this section we discuss again a variation in the degree of noise contained in the firms balance sheet information. This time, however, conditional on the optimal debt level at $t = 0$. This is to say that managers in $t = 0$ know about the degree of noise contained in future asset reports and consider the problems this causes in terms of secondary equity offerings and inability to pay when trying to refinance the then due coupon payments.

As can be seen from Figure 8, spreads decline with rising degrees of informational incompleteness. The reason is that managers choose decreasing optimal leverage with increasing noise contained in the firm's accounting reports. Optimal leverage in Figure 8 declines from 66% for the case of 1% noise contained in the asset report to 40% in case of 20% noise. This overcompensates the increasing effect that higher degrees of noise have on spreads for given levels of leverage as was seen in Figure 4. We again notice

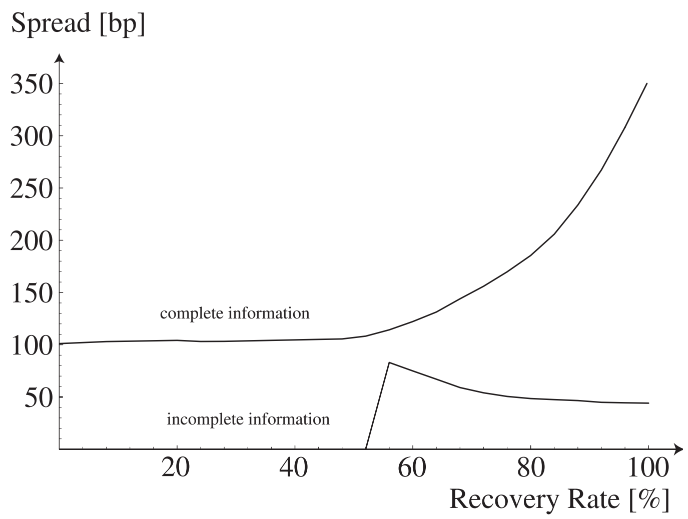

Figure 9: Spread vs. recovery rate, optimal leverage

that increasing the degree of noise loses it's effect for volatilities larger than 10%.

3.2.3 Variation of the recovery rate

Considering optimal leverage also has a significant effect on the results regarding comparativ static variation of the recovery rate. While we found that for fixed debt levels rising recovery rates lead to decreasing risk premiums both under complete and incomplete information this is not true for optimal debt levels as Figure 9 shows. With complete information higher recovery rates in case of default lead managers to assume higher leverage, thereby causing risk premiums to rise.

Under incomplete information managers also increase leverage with rising recovery rates. The increase in leverage, however, is much less than in the case of complete information and not sufficient to overcompensate the risk decreasing effects of higher recovery rates which we already showed for fixed debt levels. We also find that for low recovery rates managers choose not to assume debt under incomplete information. Only for recovery rates over 55% do managers decide to assume debt, whereby the optimal leverage jumps from 0% to 50%. This is true, since many costs of leverage under incomplete information, such as the cost of underpricing new shares and possible additional default due to inability to pay, occur with the first unit of assumed debt, whereas the benefits occur gradually. Therefore extremely low leverage as well as leverage in unfavorable conditions (e.g. low recovery rates or low tax rates) are not acceptable to managers under incomplete information, as the value of the levered firm would be lower than that of the unlevered firm. This does not occur with complete information, since here the cost of leverage rise gradually with the amount of assumed debt and positive leverage is always prefered as long as the corporate tax rate is positiv.

4 Conclusion

We were able to endogenize inability to pay as an independent default trigger into a structural model by assuming incomplete information. Rational outsiders decide on the basis of incomplete information whether or not it is worthwhile for them to participate in the firm's secondary equity offering. If they decide not to spend the amount necessary to audit the firm, the secondary placement fails and the firm is unable to pay even though it need not be overindebted. If the secondary equity placement succeeds, it will be underpriced. Underpricing is explained and necessary in this context since rational outsiders would not expend money or effort in order to be able to buy fairly priced shares.

Possible default due to inability to pay causes the values of debt and equity in the primary market to depend on the degree of informational incompleteness. This is different to the existing literature and leads to several noteworthy effects. We find that the influence of incomplete information on investors is limited as they turn to other means of information once the accounting reports become to noisy. We also find that managers not always choose positive levels of leverage under incomplete information and generally assume less debt that they would under complete information. Lower optimal leverage leads to lower spreads in the primary market for firms faced with incomplete information. This is surprising as one would expect additional risk to cause higher spreads.

Overall, incomplete information is not sufficient to explain positive short term spreads in the primary market as long as the initial asset value is known. Positive short term spreads in the secondary market, however, can be explained with incomplete information and low asset reports. Our model is also suitable to quantify the agency cost of incomplete information as the value of the levered firm is available both under complete and incomplete information. Secondary market valuation and the calculation of agency costs have been addressed and will be dealt with in subsequential papers.

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